

MATHEMATICAL MODELING AND SENSITIVITY
ANALYSIS OF RADIATION EFFECTS ON SEMI-
CONDUCTOR JUNCTIONS

by

Leon Eugene Drouin, Jr.

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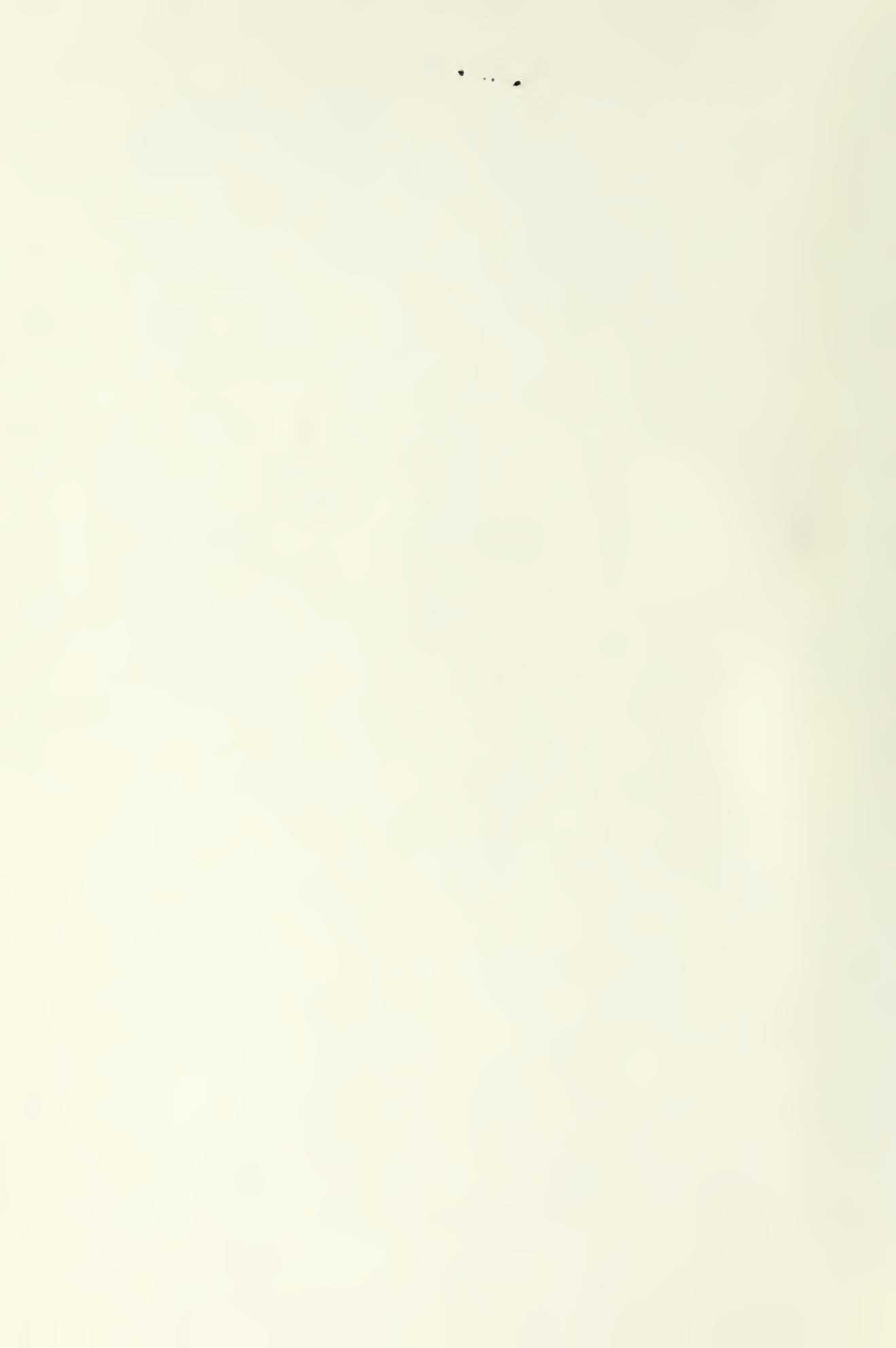
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June 1970

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Mathematical Modeling and Sensitivity Analysis
of
Radiation Effects on Semiconductor Junctions

by

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ABSTRACT

This thesis is concerned with the numerical solution of a semiconductor junction recovery from a radiation pulse. The junction is represented by an Ebers-Moll model to account for diffusion current and space-charge capacitance. The radiation pulse is considered as giving rise to a photocurrent to which it is related by a linear differential equation. Exact solutions are presented and the recovery time is presented and discussed as a function of several parameters. A simple piecewise linear analysis for a diode circuit is also presented to provide insight into the nature of the transient response and the recovery time.

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I. INTRODUCTION

The effect of a radiation pulse on a semiconductor junction is of current interest. In this thesis the transient response of a semiconductor junction to a radiation pulse is analysed and calculated using an Ebers-Moll model to account for the nonlinear diode diffusion current as well as the nonlinear space-charge capacitance. The radiation pulse is considered as giving rise to a photocurrent which acts on the junction. The photocurrent and the radiation pulse are shown, as a practical approximation, to be related through a first-order, linear differential equation.

In order to provide insight into the response, a diode circuit is first analysed using a piecewise linear approximation for the junction current and its capacitance. The analysis, although approximate, gives rise to expressions for the response and the recovery time which are useful for general purposes.

The precise transient response is then calculated for a diode and a transistor using the exact Ebers-Moll model expressions. Families of the resulting waveforms are presented for different values of circuit and junction parameters. Finally, the calculated recovery times are plotted as functions of various parameter values. The data indicates that the latter curves are generally linear over a range of parameter values such that it is possible to express recovery time as a linear function of parameters with the coefficients obtained directly from the calculated data. A sensitivity study is also made for the diode circuit to check the correctness of the coefficients obtained from the data.

II. DETERMINATION OF RADIATION PHOTOCURRENT

The radiation photocurrent, $i_{pp}(t)$, may be approximated by the solution of the differential equation

$$i_{pp}(t) + T_R \frac{d}{dt} i_{pp}(t) = A\gamma(t) \quad (2.1)$$

where T_R is a recovery time constant and A and $\gamma(t)$ are defined in Figure 2.1. The solution of (2.1) is given by

$$i_{pp}(t) = A \left(1 - e^{-t/T_R} \right) \quad \text{for } 0 \leq t \leq T \quad (2.2a)$$

$$i_{pp}(t) = A \left(1 - e^{-t/T_R} \right) e^{-(t-T)/T_R} \quad \text{for } T \leq t \quad (2.2b)$$

The numerical solution of (2.1) may be generated easily by computer and compares favorably with the error-function form of $i_{pp}(t)$ originally developed by Wirth and Rogers [2] as follows

$$i_{pp}(t) = qagL_p \left[\operatorname{erf} \left(\frac{t}{T_p} \right)^{\frac{1}{2}} - \operatorname{erf} \left(\frac{t-T}{T_p} \right)^{\frac{1}{2}} \right] \quad (2.3)$$

where q is electron charge, a is the area of the junction, g is the radiation-induced density of holes and electrons per second, L_p is the diffusion distance, and T_p is the lifetime of the minority carriers.

Figure 2.2 compares the solution of (2.1) with (2.3). Further confirmation of the use of (2.1) is obtained by favorable comparison with the results of Sigfridsson and Leman [3]. Since the integration of (2.1) can be performed easily, its use over (2.3) for numerical calculations is readily justified.

III. ANALYSIS OF THE DIODE CIRCUIT

A. DIODE JUNCTION - MATHEMATICAL MODEL

The response of the diode circuit of Figure 3.1 to the radiation pulse of Figure 2.1 is considered. The mathematical model for the diode junction is illustrated in Figure 3.2 leading to the following equations

$$[C_s(v) + C_d] \frac{dv}{dt} + Gv + I + i_d = i_{pp}(t) \quad (3.1)$$

$$G = (R_{sh} + R)/R_{sh}R \quad (3.2)$$

$$I = E/R \quad (3.3)$$

$$i_d = I_s(e^{\alpha v} - 1) \quad (3.4)$$

R_{sh} is the shunt leakage resistance of the diode and i_d is the diode diffusion current which is related to the diode voltage by (3.4), where $\alpha = q/kT$. q is electron charge, k is Boltzman's constant, and T is absolute temperature. I_s is the reverse saturation current for the junction. $C_d(v)$ and $C_s(v)$ represent the nonlinear capacitance effects of diode storage time and space charge respectively. E. Steele [1] has shown that $C_s(v)$ may be obtained from the following equation

$$i_{cs} = T_D \frac{di_d}{dt} = T_D I_s \alpha e^{\alpha v} \frac{dv}{dt} \quad (3.5)$$

where T_D is the storage time constant. From (3.5) it follows that

$$C_s(v) = T_D I_s \alpha e^{\alpha v} \quad (3.6)$$

E. Steele [1] has also shown that $C_d(v)$ depends upon the direction of the junction bias, namely

$$C_d(v) = C_o / (1 - \frac{v}{V_d})^n \quad \text{for reverse bias} \quad (3.7a)$$

$$C_d(v) = 0 \quad \text{for forward bias} \quad (3.7b)$$

where V_d is the diffusion or built-in voltage (0.75 volts for silicon) and

$n = \frac{1}{2}$ for an abrupt junction, or $1/3$ for a linear graded junction. In the voltage range of interest for this thesis, the change in $C_d(v)$ with voltage is less than two percent of the change in $C_s(v)$. It is therefore reasonable to assume that $C_d(v)$ is a constant

$$C_d(v) = C_d \quad (3.8)$$

In terms of (3.3), (3.4) and (3.5), equation (3.1) may be written in the following composite form

$$\frac{dv}{dt} = F(v) + B(v) u(t) \quad (3.9)$$

where

$$F(v) = - \frac{Gv + I_s (e^{\alpha v} - 1)}{C_D + T_D I_s \alpha e^{\alpha v}} \quad (3.10)$$

$$B(v) = 1/(C_D + T_D I_s \alpha e^{\alpha v}) \quad (3.11)$$

$$u(t) = i_{pp}(t) - E/R \quad (3.12)$$

where $i_{pp}(t)$ is obtained from the solution of (2.1).

The exact solution of (3.9) is considered as obtained using a digital computer. However, in order to provide insight into the nature of the radiation transient, it is worthwhile to first analyse the circuit assuming piecewise linearity, as discussed in the next section.

B. PIECEWISE LINEAR SOLUTION

The diode is considered to be piecewise linear so that

$$i_d = v/R_d \quad \text{for } v \geq 0 \quad (3.13a)$$

$$i_d = 0 \quad \text{for } v < 0 \quad (3.13b)$$

where $R_d \ll R \ll R_{sh}$. Also

$$C_D + C_s(v) = C_s(v) = C_s \quad \text{for } v \geq 0 \quad (3.14a)$$

$$C_D + C_s(v) = C_D \quad \text{for } v < 0 \quad (3.14b)$$

Thus both the junction and resistance and capacitance are considered to be

piecewise linear. In order to simplify the solution further, it is assumed first that $i_{pp}(t)$ is a rectangular pulse of height I_{ppo} and width T . The following numerical values are used as typical values

$$\begin{aligned} E &= 10\text{v} & R &= 10^4 \text{ ohms} \\ I_{ppo} &= 5(10)^{-3} \text{ amps} & T &= 0.1 \mu \text{ sec} \\ R_{sh} &= 10^8 \text{ ohms} & C_s &= 50 \text{ pF} \\ R_D &= 10^3 \text{ ohms} & C_D &= 4 \text{ pF} \end{aligned}$$

Figure 3.3 presents the junction response with the above piecewise linear approximations. In regions A the junction is forward biased, whereas in region B it is back biased. From the simple time constants of this approximation, it follows that

$$T_1 = \tau_1 \ln \left[\frac{I_{ppo} R}{I_{ppo} R - E} \right] \text{ where } \tau_1 = RC_D \quad (3.15a)$$

$$T_1 \cong \tau_1 \left[\frac{E}{I_{ppo} R} \right] \quad (3.15b)$$

The approximation used is discussed in Appendix A.

At the time $t = T$, the end of the photocurrent pulse, the junction voltage is given by

$$v(T) = (ER_D/R - I_{ppo} R_D) (1 - e^{-(T - T_1)/\tau_2}) \quad (3.16a)$$

$$v(T) \cong (ER_D/R - I_{ppo} R_D) \text{ when } (T - T_1) \gg \tau_2 \quad (3.16b)$$

where $\tau_2 = R_D C_s$

The time interval, T_3 , measures from the end of the photocurrent pulse to the point when the junction voltage returns to zero, is given by

$$T_3 = \tau_2 \ln \left(\frac{ER_D/R - v(T)}{ER_D/R} \right) \quad (3.17a)$$

$$T_3 \cong \tau_2 \ln \left(\frac{I_{ppo} R_D}{ER_D/R} \right) \cong \tau_2 (1 - E/I_{ppo} R) \quad (3.17b)$$

Equation (3.16b) is used for the first approximation, and the second

approximation is discussed in Appendix A.

The junction recovery time, T_{rec} , is given by

$$T_{rec} = T + T_3 + 3\tau_1 \quad (3.18a)$$

$$T_{rec} \cong T + \tau_2 [1 - (E/I_{ppo} R)] = 3\tau_1 \quad (3.18b)$$

Substituting the numerical values listed previously into (3.18b) yields

$T_{rec} = 0.26 \mu\text{sec}$, which compares favorably with the value of $0.27 \mu\text{sec}$,

90% of the final value, obtained from the curves of Figure 3.4. Figure

3.4 shows the junction response for several values of I_{ppo} as noted. In

Figure 3.5 the junction response is drawn for the piecewise linear

approximation of (3.13) and (3.14) with $i_{pp}(t)$ given by (2.2) where

$T_R = 0.2 \mu\text{sec}$ and $A = 10 \text{ ma}$. In the next section the junction response

using the nonlinear equations (3.1) through (3.6) is considered.

C. EXACT NONLINEAR SOLUTION

Using trapezoidal integration [4] over the time interval, ΔT , equation (3.9) becomes

$$v_n - v_{n-1} = \frac{\Delta T}{2} [F(v_n) + F(v_{n-1})] + \frac{\Delta T}{2} [B(v_n)u_n + B(v_{n-1})u_{n-1}] \quad (3.19)$$

where $v_n = v[n\Delta T]$, $v_{n-1} = [(n-1)\Delta T]$, $u_n = u(n\Delta T)$, and $u_{n-1} = u[(n-1)$

$\Delta T]$. Using the first two terms of a Taylor series expansion to express

$F(v_n)$ and $B(v_n)$ in terms of $F(v_{n-1})$ and $B(v_{n-1})$ respectively, enables

(3.19) to be written as

$$v_n = v_{n-1} + \frac{\Delta T [F(v_{n-1}) + \frac{1}{2}(u_n + u_{n-1}) B(v_{n-1})]}{1 - \frac{\Delta T}{2} \left[\frac{\partial}{\partial v} F(v) + u_n \frac{\partial}{\partial v} B(v) \right] v_{n-1}} \quad (3.20)$$

where the partial derivatives are obtained by direct calculation from

(3.10), (3.11) and (3.12). Equation (3.20) is recursive and is easily

programmed. In the results presented, the time interval, ΔT , is adjusted

automatically such that

$$|v_n - v_{n-1}| \leq 0.01 \text{ volts} \quad (3.21)$$

The numerical results obtained have been confirmed by comparison with a lengthier integration scheme using fourth order Runge Kutta [5] integration.

The nominal values used for numerical calculations are listed below. These values are typical of junctions which have been produced and measured at the Autonetics Division of North American Rockwell Corporation, Anaheim, California.

$$\begin{array}{lll}
 E = 10 \text{ volts} & R = 10 \text{ kohms} & C_D = 4 \mu\mu\text{F} \\
 A = 5 \text{ ma} & T_R = 0.2 \mu\text{sec} & T_D = 0.05 \mu\text{sec} \\
 I_s = 1 \mu\mu\text{amp} & R_{sh} = 100 \text{ Mohms} &
 \end{array}$$

Numerical results are presented in Figures 3.6 through 3.10 for various values of A, R, E, T_R , and C_D respectively.

The dependence of the diode recovery time upon various parameters is summarized in Figures 3.11 through 3.15. Recovery time is defined as the time for the diode voltage to recover to 90% of its original quiescent value. In each of these curves, all other parameters are held at the nominal values listed previously. It is significant to note that the curves are essentially linear, with the exception of Figure 3.15 which presents recovery time as a function of A. If A is confined between the values of 5 ma and 10 ma, this curve can also be considered linear. The computer solution for this problem is presented in Appendix B.

D. LINEAR APPROXIMATION

From the linear curves it is possible to express the recovery time as a linear function of the parameters as follows:

$$T_{rec} = k_1 T_R + k_2 R + k_3 C_D + k_4 A + k_5 E + k_6 \quad (3.22)$$

where T_R is in μsec , C_D is in $\mu\mu\text{F}$, E is in volts, R is in kohms, A is in mas, and T_{rec} is in μsec . The data presented gives rise to the following constants

$$k_1 = 2.38 \mu s / \mu s$$

$$k_4 = 0.0276 \mu s / \text{ma}$$

$$k_2 = 0.027 \mu s / \text{kohm}$$

$$k_5 = -0.02 \mu s / \text{volt}$$

$$k_3 = 0.013 \mu s / \text{pf}$$

$$k_6 = 0.006 \mu s$$

With these values, (3.22) approximates the calculated data within 2% for $7(10)^3 \leq R \leq 12(10)^3$; $1\mu\mu\text{F} \leq C_D \leq 9\mu\mu\text{F}$; $5 \text{ ma} \leq A \leq 10 \text{ ma}$; $0.1 \mu\text{sec} \leq T_R \leq 0.25 \mu\text{sec}$ and $9\text{V} \leq E \leq 11\text{V}$. The coefficients of (3.22) are a direct measure of the relative sensitivities of recovery time to the several parameters.

In conclusion it should be noted that, in addition to the specific results presented, the use of the mathematical model and numerical integration is confirmed as a reasonable approach for the analysis of radiation transients in semiconductors. In Chapter V the extension of this approach to transistor circuits is considered.

IV SENSITIVITY ANALYSIS OF NONLINEAR CIRCUITS

A. INTRODUCTION

An sensitivity analysis of general nonlinear circuits is developed in this chapter, and the results are applied to the diode problem of Chapter II, thus indicating the validity of the coefficients of (3.22). The sensitivity study is based upon the work of S. R. Parker [6] and R. Lee [7], who utilized the concept of auxiliary coupled networks applied to the state equation format adopted previously.

B. SENSITIVITY AUGMENTED STATE EQUATION

The general state equation for a nonlinear circuit can be written as

$$\dot{\underline{x}}(t) = \underline{A}[\underline{x}(t)] \underline{x}(t) + \underline{B}[\underline{x}(t)] \underline{u}(t) + \underline{c}[\underline{x}(t)] \quad (4.1a)$$

or

$$\dot{\underline{x}}(t) = \underline{f}(\underline{A}, \underline{c}) + \underline{B}[\underline{x}(t)] \underline{u}(t) \quad (4.1b)$$

where

$$\underline{f}(\underline{A}, \underline{c}) = \underline{A}[\underline{x}(t)] \underline{x}(t) + \underline{c}[\underline{x}(t)] \quad (4.1c)$$

The sensitivity function defined by S. R. Parker [6]

$$S_{a_j}^{x_i} = \frac{\partial x_i}{\partial \ln a_j} \quad \text{for } i = 1, 2, \dots, n \text{ and } j = 1, 2, \dots, r \quad (4.2)$$

This definition for the sensitivity function enables a convenient comparison of the sensitivity functions on a percentage basis when the parameters being studied have widely different nominal values.

The sensitivity function may be obtained by performing the indicated partial differentiations on (4.1). Combining the results of the differentiation with (4.1) yields the sensitivity augmented state equation

$$\begin{vmatrix} \dot{\underline{x}} \\ \underline{x}_{s_j} \end{vmatrix} = \begin{vmatrix} \underline{A} & \underline{0} \\ \underline{0} & \underline{A}_D \end{vmatrix} \begin{vmatrix} \underline{x} \\ \underline{x}_{s_j} \end{vmatrix} + \begin{vmatrix} \underline{B} & \underline{0} \\ \underline{0} & \underline{B}_D \end{vmatrix} \begin{vmatrix} \underline{u} \\ \underline{u}_{s_j} \end{vmatrix} + \begin{vmatrix} \underline{0} & \underline{0} \\ \underline{A}_{s_j} & \underline{B}_{s_j} \end{vmatrix} \begin{vmatrix} \underline{x} \\ \underline{u} \end{vmatrix} + \begin{vmatrix} \underline{c} \\ \underline{c}_{s_j} \end{vmatrix} \quad (4.3a)$$

where

$$\dot{\underline{x}}_{sj} = \frac{\partial}{\partial \ln a_j} [\underline{x}]^T \quad (4.3b)$$

$$\underline{u}_{sj} = \frac{\partial}{\partial \ln a_j} [\underline{u}]^T \quad (4.3c)$$

$$\underline{c}_{sj} = \frac{\partial}{\partial \ln a_j} [\underline{c}]^T \quad (4.3d)$$

$$\underline{A}_{sj} = \frac{\partial}{\partial \ln a_j} \underline{A} \quad (4.3e)$$

$$\underline{A}_D = \begin{vmatrix} \underline{A} & & \\ & \underline{A} & \\ & & \underline{A} \end{vmatrix} \quad (4.3g)$$

$$\underline{B}_D = \begin{vmatrix} \underline{B} & & \\ & \underline{B} & \\ & & \underline{B} \end{vmatrix} \quad (4.3h)$$

Removing the partitioned set of first-order partial differential equations yields

$$\dot{\underline{x}}_{sj} = \underline{A} \underline{x}_{sj} + \underline{c}_{sj} + \underline{A}_{sj} \underline{x} + \underline{B} \underline{u}_{sj} + \underline{B}_{sj} \underline{u} \quad (4.4a)$$

which may be rewritten as

$$\dot{\underline{x}}_{sj} = \underline{f}_{sj}(\underline{A}, \underline{x}_{sj}, \underline{c}_{sj}, \underline{A}_{sj}, \underline{x}) + \underline{B} \underline{u}_{sj} + \underline{B}_{sj} \underline{u} \quad (4.4b)$$

where

$$\underline{f}_{sj} = \underline{A} \underline{x}_{sj} + \underline{c}_{sj} + \underline{A}_{sj} \underline{x} \quad (4.4c)$$

The iterative solution to (4.4) is shown by R. Lee [7] to be

$$\begin{aligned} \underline{x}_{sj}(n) = & \underline{x}_{sj}(n-1) + \Delta T \left\{ \underline{I} - \frac{\Delta T}{2} \left[\left(\nabla_{\underline{x}_{sj}} \underline{f}_{sj}^T \right)^T + \right. \right. \\ & \left. \left(\nabla_{\underline{x}_{sj}} [\underline{B}_{sj} \underline{u}(n)]^T \right)^T \right] \}^{-1} \cdot \left[\underline{f}_{sj} + \frac{1}{2} \left(\underline{\Delta x}^T \nabla_{\underline{x}} \right) \underline{f}_{sj} + \underline{B} [\underline{u}_{sj}(n-1) + \right. \\ & \underline{u}_{sj}(n)] + \left(\underline{\Delta x}^T \nabla_{\underline{x}} \right) [\underline{B} \underline{u}_{sj}(n)] + \underline{B}_{sj} [\underline{u}(n) + \underline{u}(n-1)] + \left(\underline{\Delta x}^T \nabla_{\underline{x}} \right) \\ & \left. \left. [\underline{B}_{sj} \underline{u}(n)] \right] \right] \end{aligned} \quad (4.5a)$$

where

$$(\underline{\Delta x}^T \underline{\nabla}_{\underline{x}}) \underline{f} = (\underline{\nabla}_{\underline{x}} \underline{f}^T)^T \underline{\Delta x} \quad (4.5b)$$

and

$$\underline{\Delta x} = [\Delta x_1 \quad \Delta x_2 \quad \Delta x_3 \quad \dots \quad \Delta x_n]^T \quad (4.5c)$$

$$\underline{\nabla}_{\underline{x}} = \left[\frac{\partial}{\partial x_1} \quad \frac{\partial}{\partial x_2} \quad \frac{\partial}{\partial x_3} \quad \dots \quad \frac{\partial}{\partial x_n} \right]^T \quad (4.5d)$$

$$\underline{\nabla}_{\underline{x}_{sj}} = \left[\frac{\partial}{\partial x_{s1}} \quad \frac{\partial}{\partial x_{s2}} \quad \frac{\partial}{\partial x_{s3}} \quad \dots \quad \frac{\partial}{\partial x_{sr}} \right]^T \quad (4.5e)$$

The solution to (4.1) can easily be shown to be

$$\underline{x}(n) = \underline{x}(n-1) + \Delta T \left\{ \underline{I} - \frac{\Delta T}{2} \left[(\underline{\nabla}_{\underline{x}} \underline{f}^T)^T + (\underline{\nabla}_{\underline{x}} [\underline{B} \underline{u}(n)])^T \right]^T \right\}^{-1} \left(\underline{f} + \frac{1}{2} \underline{B} [\underline{u}(n) + \underline{u}(n-1)] \right) \quad (4.6a)$$

where

$$\underline{f} = \underline{A}[\underline{x}(t)] \underline{x}(t) + \underline{c}[\underline{x}(t)] \quad (4.6b)$$

Equations (4.5) and (4.6) can be solved directly for $\underline{x}(n)$ and $\underline{x}_{sj}(n)$ from the known values of $\underline{x}(n-1)$, $\underline{u}(n)$, $\underline{u}(n-1)$, $\underline{u}_{sj}(n-1)$, and $\underline{u}_{sj}(n-1)$, and $\underline{u}_{sj}(n-1)$. The solutions are accomplished simultaneously and yield the time responses of the states as well as their corresponding sensitivities.

C. SENSITIVITY ANALYSIS OF THE DIODE CIRCUIT

In this section the above approach is applied to the diode circuit of Chapter III. The state equation from (3.9) is

$$\dot{v}(t) - F(v) + B(v) u(t) \quad (4.7)$$

where $F(v)$, $B(v)$ and $u(t)$ are given by (3.10), (3.11) and (3.12)

respectively. As there is only one state the solution (4.5) reduces to

$$v_{sj}(n) = v_{sj}(n-1) + \Delta T \left[f_{sj} + \frac{1}{2} \left(\frac{\partial f_{sj}}{\partial v} \right) \Delta v + B[u_{sj}(n) + u_{sj}(n-1)] + \frac{\partial}{\partial v} [B u_{sj}(n)] \Delta v + B_{sj} [u(n) + u(n-1)] + \frac{\partial}{\partial v} [B_{sj} u(n)] \Delta v \right] \left[1 - \frac{\Delta T}{2} \left(\frac{\partial f_{sj}}{\partial v_{sj}} + \frac{\partial}{\partial v_{sj}} [B_{sj} u(n)] \right) \right]^{-1} \quad (4.8)$$

where f_{sj} , $\frac{\partial f_{sj}}{\partial v_{sj}}$, B , B_{sj} , $\frac{\partial}{\partial v}[B u_{sj}(n)]$ and $\frac{\partial}{\partial v}[B_{sj} u(n)]$ are evaluated at the point $v(n-1)$. F_{aj} , B_{sj} , and u_{sj} for the six parameters of interest are listed in Table I.

The solution for $v(n)$ is given by (3.20). All values are as listed previously.

The computer program is presented in Appendix B, and the resulting sensitivity functions are presented in Figures 4.1 through 4.6 inclusive for the parameters T_D , E , R , A , C_D and T_R respectively.

D. INTERPRETATION OF THE SENSITIVITY FUNCTIONS

The sensitivity function is defined as

$$S_{a_j}^{x_i} = \frac{\partial x_i}{\partial \ln a_j} = \frac{\partial x_i}{\partial a_j / a_j} \quad (4.9)$$

For very small fractional changes in a parameter, $(\Delta a_j / a_j)$, (4.9) may be approximated as

$$S_{a_j}^{x_i} \doteq \Delta x_i / (\Delta a_j / a_j) \quad (4.10)$$

Changes in the response due to changes in the value of the parameter may be approximated by solving (4.10) for Δx_i as follows:

$$\Delta x_i = S_{a_j}^{x_i} (\Delta a_j / a_j) \quad (4.11)$$

Equation (4.11) yields excellent results only when the sensitivity function is not fluctuating. When these fluctuations occur, a time shift in the response is implied. This time delay may be approximated by

$$T_{\text{delay}} \doteq \frac{(\Delta a_j / a_j)}{x_{i\text{ss}}} \int S_{a_j}^{x_i} dt \quad (4.12)$$

where $x_{i\text{ss}}$ is the steady-state value of x_i , and the integral implies the total area under the fluctuating portion of the sensitivity function. To

| $S_{a_j}^v$ | f_{sj} | B_{sj} | u_{sj} |
|--|--|--|---|
| $v_{s1} = \frac{\partial v}{\partial \ln T_D}$ | $-\frac{(G + k_1)v_{s1}}{D} + \frac{T_D k_1 (1 + \alpha v_{s1})(Cv + k_2)}{D^2}$ | $-\frac{T_D k_1 (1 + \alpha v_{s1})}{D^2}$ | 0 |
| $v_{s2} = \frac{\partial v}{\partial \ln E}$ | $-\frac{(G + k_1)v_{s2}}{D} + \frac{\alpha T_D k_1 v_{s2} (Gv + k_2)}{D^2}$ | $-\frac{\alpha T_D k_1 v_{s2}}{D^2}$ | $-\frac{E}{R}$ |
| $v_{s3} = \frac{\partial v}{\partial \ln R}$ | $\frac{(G + k_1)v_{s3} + \frac{v}{R}}{D} + \frac{\alpha T_D k_1 v_{s3} (Gv + k_2)}{D^2}$ | $-\frac{\alpha T_D k_1 v_{s3}}{D^2}$ | $\frac{E}{R}$ |
| $v_{s4} = \frac{\partial v}{\partial \ln A}$ | $-\frac{(G + k_1)v_{s4}}{D} + \frac{\alpha T_D k_1 v_{s4} (Cv + k_2)}{D^2}$ | $-\frac{\alpha T_D k_1 v_{s4}}{D^2}$ | $\frac{\partial i_{pp}(t)}{\partial \ln A}$ |
| $v_{s5} = \frac{\partial v}{\partial \ln C_D}$ | $-\frac{(G + k_1)v_{s5}}{D} + \frac{(C_D + \alpha T_D k_1 v_{s5})(Gv + k_2)}{D^2}$ | $-\frac{C_D + \alpha T_D k_1 v_{s5}}{D^2}$ | 0 |
| $v_{s6} = \frac{\partial v}{\partial \ln T_R}$ | $-\frac{(G + k_1)v_{s6}}{D} + \frac{\alpha T_D k_1 v_{s6} (Gv + k_2)}{D^2}$ | $-\frac{\alpha T_D k_1 v_{s6}}{D^2}$ | $\frac{\partial i_{pp}(t)}{\partial \ln T_R}$ |

$$k_1 = I_S \alpha e^{\alpha v}$$

$$k_2 = I_S (e^{\alpha v} - 1)$$

$$D = C_D + T_D k_1$$

$S_{a_j}^v$ = sensitivity function of "v" with respect to "a_j".

$$\underline{f}_{sj} = \underline{A} \underline{x}_{sj} + \underline{c}_{sj} + \underline{A}_{sj} \underline{x}, \quad \underline{u}_{sj} = \frac{\partial}{\partial \ln a_j} [\underline{u}]^T$$

$$\underline{B}_{sj} = \frac{\partial}{\partial \ln a_j} \underline{B}$$

TABLE I. FUNCTIONS NEEDED FOR THE SOLUTION OF (4.8).

apply (4.12) to the diode problem results shown in Figures 4.1 through 4.6 inclusive, (4.12) can be written as

$$\frac{\Delta T_{rec}}{\Delta a_j} = \frac{1}{a_j x_{iss}} \int S_{a_j}^{x_i} dt \quad (4.13)$$

This form of the equation will give a direct comparison with the coefficients of (3.22).

The results and the comparison with the coefficients of (3.22) are shown in Table II. The area under the sensitivity functions was found by utilizing trapezoidal integration techniques.

| <u>j</u> | <u>a_j</u> | <u>∫ S_{a_j}^v</u> | <u>$\frac{\Delta T_{rec}}{\Delta a_j}$</u> | <u>k_j</u> |
|----------|-------------------------|--|---|----------------------|
| 1 | T _R = 0.2 μs | 4.307 v μs | 2.154 μs/μs | 2.38 μs/μs |
| 2 | R = 10 kohms | 2.693 v μs | 0.0269 μs/kohm | 0.027 μs/kohm |
| 3 | C _D = 4 μμF | 0.552 v μs | 0.0138 μs/μμF | 0.013 μs/μμF |
| 4 | A = 10 ma | 2.597 v μs | 0.0260 μs/ma | 0.0276 μs/ma |
| 5 | E = 10 v | -2.050 v μs | -0.0205 μs/v | -0.02 μs/v |
| 6 | T _D = 0.05ns | 0.124 v μs | 0.0013 μs/μs | 0.0 μs/μs |

v_{ss} = 10 volts, steady-state voltage

a_j = parameter of interest

∫ S_{a_j}^v = area under the fluctuating part of the sensitivity function

$$\frac{\Delta T_{rec}}{\Delta a_j} = \frac{1}{a_j v_{ss}} \int S_{a_j}^{x_i} dt$$

k_j = the coefficients of (3.22)

TABLE II. COMPARISON OF SENSITIVITY FUNCTION RESULTS AND LINEAR COEFFICIENTS.

The results indicated in Table II show that the previously calculated

coefficients compare very favorably with those generated by the sensitivity function approach to the nonlinear circuit. Since the previous results were derived from linear approximations, it is assumed that the sensitivity function approach yields the more accurate results. The accuracy of the sensitivity function solution is limited only by the accuracy of the computation of the area under the sensitivity function.

V. ANALYSIS OF THE TRANSISTOR CIRCUIT

A. TRANSISTOR CURCUIT - MATHEMATICAL MODEL

In this chapter the effects of radiation current pulses injected across the emmitter and collector junctions of a n-p-n transistor operating in the active region are considered. The separate effects of the current pulses injected across each of the junctions is also considered. The circuit is shown in Figure 5.1.

The radiation pulses are defined by (2.1). The equivalent circuit based on the Ebers-Moll model with the introduction of the current pulses, voltage source and biasing resistors is shown in Figure 5.2 where

$$i_{de}(v_e) = I_{ES} (e^{\alpha v_e} - 1) \quad (5.1a)$$

$$i_{dc}(v_c) = I_{CS} (e^{\alpha v_c} - 1) \quad (5.1b)$$

$$C_{SC}(v_c) = T_{DC} I_C \alpha e^{\alpha v_c} \quad (5.1c)$$

$$C_{SE}(v_e) = T_{DE} I_E \alpha e^{\alpha v_e} \quad (5.1d)$$

$$i_{ppe}(t) = i_{ppc}(t) = i_{pp}(t) \quad (5.1e)$$

and $i_{pp}(t)$ is defined as in the diode case.

B. EXACT NONLINEAR SOLUTION

The state equations obtained by summing the currents at the collector and emmitter nodes are

$$[C_{SE}(v_e) + C_{DE}] \dot{v} + \frac{v_e}{R_{SHE}} + i_{de}(v_e) - \alpha I_{dc}(v_c) - i_{ppe}(t) - \frac{1}{R_C} (v_c - v_e + V_{CC}) = 0 \quad (5.2)$$

$$[C_{SC}(v_c) + C_{DC}] \dot{v}_c + \frac{v_c}{R_{SHC}} + i_{dc}(v_c) - \alpha_N i_{de}(v_e) - i_{ppc}(t) + \frac{1}{R_C} (v_c - v_e + V_{CC}) + \frac{v_c}{R_B} = 0 \quad (5.3)$$

Solving (5.2) and (5.3) for \dot{v}_e and \dot{v}_c respectively, and letting $x_1 = v_e$ and $x_2 = v_c$, yields the general form of the state equation for nonlinear circuits

$$\dot{\underline{x}}(t) = \underline{A}[\underline{x}(t)] \underline{x}(t) + \underline{B}[\underline{x}(t)] \underline{u}(t) + \underline{C}[\underline{x}(t)] \quad (4.1)$$

which for the case of interest can be written

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \quad (5.4a)$$

where

$$A_{11} = -\left(\frac{1}{R_C} + \frac{1}{R_{SHE}}\right) / [C_{SE}(x_1) + C_{DE}] \quad (5.4b)$$

$$A_{12} = 1 / [C_{SE}(x_1) + C_{DE}] R_C \quad (5.4c)$$

$$A_{21} = 1 / [C_{SC}(x_2) + C_{DC}] R_C \quad (5.4d)$$

$$A_{22} = -\left(\frac{1}{R_B} + \frac{1}{R_C} + \frac{1}{R_{SHC}}\right) / [C_{SC}(x_2) + C_{DC}] \quad (5.4e)$$

$$C_1 = [\alpha_I i_{dc}(x_2) - i_{de}(x_1)] / [C_{SE}(x_1) + C_{DE}] \quad (5.4f)$$

$$C_2 = [\alpha_N i_{de}(x_1) - i_{dc}(x_2)] / [C_{SC}(x_2) + C_{DC}] \quad (5.4g)$$

$$u_1 = i_{pp}(t) \quad (5.4h)$$

$$u_2 = V_{CC} \quad (5.4i)$$

$$B_{12} = 1 / [C_{SE}(x_1) + C_{DE}] R_C \quad (5.4j)$$

$$B_{21} = -1 / [C_{SC}(x_2) + C_{DC}] R_C \quad (5.4k)$$

and $B_{11} = 1 / [C_{SE}(x_1) + C_{DE}]$ for $i_{ppe}(t) \neq 0$ (5.4l)

$$B_{11} = 0 \quad \text{for } i_{ppe}(t) = 0$$

$$B_{21} = 1 / [C_{SC}(x_2) + C_{DC}] \quad \text{for } i_{ppc}(t) \neq 0 \quad (5.4m)$$

$$B_{21} = 0 \quad \text{for } i_{ppc}(t) = 0 \quad (5.4m)$$

The recursive solution to (5.4) is given by (4.6). The computer program for the solution is presented in Appendix B.

The typical parameter values used are as follows

| | | |
|-------------------------------|-------------------------------|---------------------------------|
| $\alpha_N = 1.0$ | $I_{SC} = 0.01 \text{ na}$ | $T_{DE} = 1.0 \text{ ns}$ |
| $\alpha_I = 0.99$ | $I_{SE} = 0.1 \mu\text{a}$ | $T_{DC} = 0.1 \mu\text{s}$ |
| $R_{SHC} = 100 \text{ Mohms}$ | $C_{DE} = 1.0 \mu\mu\text{F}$ | $\alpha = 38.46 \text{ v}^{-1}$ |
| $R_{SHE} = 100 \text{ Mohms}$ | $C_{DC} = 1.0 \mu\mu\text{F}$ | $V_{CC} = 10 \text{ v}$ |
| $R_B = 0.35 \text{ Mohms}$ | $R_C = 4.5 \text{ kohms}$ | $T = 1.0 \mu\text{s}$ |

which yield the static conditions $v_e = x_1 = 0.6 \text{ v}$ and $v_c = x_2 = -3.4 \text{ v}$. Typical output responses are shown in Figure 5.3 for variations in the amplitude of the radiation pulse.

C. LINEAR APPROXIMATION

The dependence of the transistor circuit recovery time upon the parameters T_R , C_{DC} , C_{DE} , T_{DC} , T_{DE} and A is presented in Figures 5.4 through 5.7. In each of these curves, all other parameters are held constant at the nominal values listed above. It is significant to note, as it was in the diode problem, that the curves are essentially linear with the exception of Figure 5.7 which represents the recovery time as a function of A . If A is confined between the values of 0.01ma and 0.2ma this curve can also be considered linear. Note that these restrictions are not improper as slight variations around the nominal values are of interest.

From the linear curves it is possible to express the recovery time as a linear function of the parameters as follows:

$$T_{rec} = k_1 T_R + k_2 C_{DC} + k_3 C_{DE} + k_4 T_{DC} + k_5 T_{DE} + k_6 A + k_7 \quad (5.5)$$

The values of the coefficients of (5.5) are presented in Table III.

| | $i_{ppc}(t) = 0$ | $i_{ppe}(t)=0$ | $i_{ppc}(t) = i_{ppe}(t) = i_{pp}(t)$ |
|-------|------------------------|------------------------|---------------------------------------|
| k_1 | 0.0 | 0.0 | 0.0 |
| k_2 | +0.22 $\mu s/\mu\mu F$ | +0.24 $\mu s/\mu\mu F$ | +0.275 $\mu s/\mu\mu F$ |
| k_3 | 0.0 | 0.0 | 0.0 |
| k_4 | +7.20 $\mu s/\mu s$ | +7.20 $\mu s/\mu s$ | +10.25 $\mu s/\mu s$ |
| k_5 | 0.0 | 0.0 | 0.0 |
| k_6 | +39.6 $\mu s/ma$ | +36.8 $\mu s/ma$ | +65.3 $\mu s/ma$ |
| k_7 | -0.2 μs | +0.07 μs | -0.05 μs |

$i_{pp}(t)$ = radiation photocurrent pulse defined by (2.2).

TABLE III. COEFFICIENTS OF (5.5)

With the values listed in Table III, (5.5) approximates the calculated data within 3% for $0.5 \mu\mu F \leq C_{DC} \leq 1.5 \mu\mu F$; $0.05 \mu s \leq T_{DC} \leq 0.15 \mu s$; $0.03 ma \leq A \leq 0.15 ma$; $0.1 \mu s \leq T_R \leq 4.0 \mu s$; $0.05 \mu s \leq T_{DE} \leq 4 \mu s$ and $0.5 \mu\mu F \leq C_{DE} \leq 4.0 \mu\mu F$.

VI. CONCLUSIONS

The radiation pulse gives rise to a photocurrent which causes the junction to be temporarily back biased. The recovery time, for small ranges in the parameters of the circuit and the radiation pulse, is a linear function of the parameters. The sensitivity analysis approach yields the coefficients of the linear function directly, whereas it is necessary to make several calculations, using different values of the parameters, to get sufficient data utilizing the straightforward state space approach.

APPENDIX A

TIME CONSTANT APPROXIMATION

The approximation for the time constants in Chapter III is based upon the following. Consider the exponential function

$$y = x e^{-T/\beta} . \tag{1}$$

Then

$$T = \beta \ln\left(\frac{x}{y}\right); \quad x \geq y. \tag{2}$$

If $T/\beta \ll 1$, then (1) becomes

$$y \doteq x (1 - T/\beta) \tag{3}$$

and

$$T \doteq \beta(1 - y/x) = \beta\left(\frac{x - y}{x}\right); \quad x \geq y. \tag{4}$$

APPENDIX B

```

C   TRANSISTOR SOLUTION-NONLINEAR STATE VARIABLE FORMAT
      REAL*8  ITITLE
      DIMENSION F(2,1),DER(2,2),AI(2,2),TEMP(2,2),AIPP(3005)
      1V(3005),XP(3005),XS(2,1),TEMP1(2,2),DERINV(2,2),
      2B(2,2),U(2,1),BDER(2,2),UT(2,1),TEMP2(2,2),UTEMP(2,1)
      3,ITITLE(12)
      1  FORMAT('1',//,7X,'RADC',11X,'TR',13X,'TDE',12X,'TDC',1
      1,'CDE')
      2  FORMAT(6E10.2)
      3  FORMAT(6E18.6)
      10 FORMAT(/,6E15.6)
      11 FORMAT(6A8)
      12 FORMAT(A4)
      13 FORMAT(/,1X,'L=',I10)
      14 FORMAT('1',//,9X,'TI',15X,'U(1,1)',14X,'VE',16X,'VC',
      115X,'V(TI)',15X,'T')
      15 FORMAT(I10,3E10.2)
      20 FORMAT(8E10.2)
      CALL CANCEL(2)

```

```

C
      NS=2
      NU=1
      READ(5,11) ITITLE
      READ(5,12) LABEL1
      READ(5,20) ((AI(I,J),J=1,NS),I=1,NS)
      READ(5,2) RSHE,CDE,RB ,SIE,ALPE,ALPI
      READ(5,2) RSHC,CDC,RC ,SIC,ALPC,ALPN
      READ(5,2) E,TDC,TDE,T,TR,RADC
      25 N=1
      C   INSERT CONTROL CARDS FOR IPPE=0, KK=1, FOR IPPC=0,
      C   KK=2, FOR BCTH PULSES KK=3,,KK=4 ENDS COMPUTATIONS
      READ(5,15) KK,TDC,CDE,CDC
      WRITE(6,1)
      WRITE(6,10) RADC,TR,TDE,TDC,CDC,CDE
      WRITE(6,14)

```

```

C
      TAU= 1.0E-06
      TT= 1.2*TAU
      TI=0.
      TMAX=1.0E-08
      XMIN= 0.05
      HCNT=1.
      X1=.6
      X2=-3.4
      V(1)=X1-X2
      XP(1)=TI
      AIPP(1)= 0.0
      XS(1,1)=X1
      XS(2,1)=X2
      UTEMP(1,1)=0.
      UTEMP(2,1)=E
      VV= 0.9*V(1)
      GE=1./RC+1./RSHE
      GC=1./RC+1./RSHC+1./RB
      Z= RADC*( 1.0-EXP(-TAU/TR))
      6  CONE1=SIE*ALPE*EXP(ALPE*X1)
      CONE2=TDE*CCNE1
      CONE3=ALPE*CONE2
      CONE4=SIE*(EXP(ALPE*X1)-1.)
      DENE=CDE+CCNE2
      CONC1=SIC*ALPC*EXP(ALPC*X2)
      CONC2=TDC*CCNC1
      CONC3=ALPC*CONC2
      CONC4=SIC*(EXP(ALPC*X2)-1.)
      DENC=CDC+CONC2
      F(1,1)=(-GE*X1+X2/RC+ALPI*CONC4-CONE4)/DENE
      F(2,1)=(-GC*X2+X1/RC+ALPN*CONE4-CONC4)/DENC
      IF(KK.EQ.1) GO TO 60
      B(1,1)=1./DENE

```



```

GO TO 61
60 B(1,1)=0.
61 IF(KK.EQ.2) GO TO 62
   B(2,1)=1.0/DENC
   GO TO 63
62 B(2,1)=0.
63 CONTINUE
   B(1,2)=1.0/(DENE*RC)
   B(2,2)=-1.0/(DENC*RC)
   U(2,1)=E
   DER(1,1)=- (GE+CONE1+CONE3*F(1,1))/DENE
   DER(1,2)=(1.0/RC+ALPI*CONC1)/DENE
   DER(2,1)=(1.0/RC+ALPN*CONC1)/DENC
   DER(2,2)=- (GC+CONC1+CONC3*F(2,1))/DENC
8 IF(TI.LT.TAU) GO TO 4
   U(1,1)= Z*EXP((-TI+TAU)/TR)
   GO TO 9
4 U(1,1)= RADC*(1.0-EXP(-TI/TR))
9 CONTINUE
   IF(KK.EQ.1) GO TO 50
   BDER(1,1)=- (CONE3*(U(1,1)+E/RC))/(DENE**2)
   GO TO 51
50 BDER(1,1)=- (CONE3*E)/(RC*(DENE**2))
51 IF(KK.EQ.2) GO TO 52
   BDER(2,2)=- (CONC3*(U(1,1)-E/RC))/(DENC**2)
   GO TO 53
52 BDER(2,2)= (CONC3*E)/(RC*(DENC**2))
53 CONTINUE
   BDER(1,2)=0.
   BDER(2,1)=0.
   CALL ADD (BDER,DER,NS,NS,TEMP1)
   Q=T/2.
   CALL CONST (Q,TEMP1,NS,NS,TEMP)
   CALL SUB (AI,TEMP,NS,NS,TEMP1)
   CALL RECIP (.000001,TEMP1,DERINV,KER,NS)
   Q=T
   CALL CONST (Q,DERINV,NS,NS,TEMP)
   CALL ADD (U,UTEMP,NS,NU,UT)
   Q=.5
   CALL CONST (Q,UT,NS,NU,UT)
   CALL PROD (B,UT,NS,NS,NU,TEMP2)
   CALL ADD (TEMP2,F,NS,NU,TEMP1)
   CALL PROD (TEMP,TEMP1,NS,NS,NU,TEMP2)
   CALL ADD (TEMP2,XS,NS,NU,TEMP)
   DELX1=TEMP(1,1)-XS(1,1)
   DELX2=TEMP(2,1)-XS(2,1)
   XNORM=SQRT(DELX1**2+DELX2**2)
   IF(XNORM.GT.XMIN) GO TO 5
   GO TO 7
5 T=T/2.
   TI=TI-T
   IF(TI.LE.0.0) TI= 0.0
   GO TO 8
7 X1=TEMP(1,1)
   X2=TEMP(2,1)
   Q=1.
   CALL CONST (Q,U,NS,NU,UTEMP)
   CALL CONST (Q,TEMP,NS,NU,XS)
   TEMT=HCNT*1.0E-08
   HCNT=HCNT+1.
   XP(N)=TI
   V(N)=X1-X2
   AIPP(N)=U(1,1)*V(1)/RADC
   WRITE(6,3) TI,U(1,1),X1,X2,V(N),T
   IF(TI.GT.TT) TMAX= 5.0E-08
   T=TMAX
   TI=TI+T
   VVV=V(N)
   N=N+1
   IF(N.GE.900) GO TO 31
   IF(N.LT.100) GO TO 6
29 IF(VVV.LT.VV) GO TO 6

```



```

31 CONTINUE
   N=N-1
   CALL DRAW(N,XP,V,0,0,LABEL1,ITITLE,4.0E-06,1.0,0,0,2,2,
15,5,1,L)
   WRITE(6,13) L
   IF(KK. LE.3) GO TO 25
   RETURN
   END

```

```

C   SUBROUTINE ADD (A,B,N,M,C)
   THIS SUBROUTINE ADDS TWO NXM MATRICES
   DIMENSION A(N,M),B(N,M),C(N,M)
   DO 1 I=1,N
   DO 1 J=1,M
1  C(I,J)=A(I,J)+B(I,J)
   RETURN
   END

```

```

C   SUBROUTINE SUB (A,B,N,M,C)
   THIS SUBROUTINE SUBTRACTS TWO NXM MATRICES
   DIMENSION A(N,M),B(N,M),C(N,M)
   DO 1 I=1,N
   DO 1 J=1,M
1  C(I,J)=A(I,J)-B(I,J)
   RETURN
   END

```

```

C   SUBROUTINE PROD (A,B,N,M,L,C)
   THIS SUBROUTINE POST MULTIPLIES A NXM MATRIX
   BY A MXL MATRIX
   DIMENSION A(N,M),B(M,L),C(N,L)
   DO 1 I=1,N
   DO 1 J=1,L
1  C(I,J)=0.0
   DO 2 I=1,N
   DO 2 J=1,L
   DO 2 K=1,M
2  C(I,J)=C(I,J)+A(I,K)*B(K,J)
   RETURN
   END

```

```

C   SUBROUTINE CONST(Q,A,N,M,C)
   THIS SUBROUTINE MULTIPLIES A NXM MATRIX BY A CONSTANT
   DIMENSION A(N,M),C(N,M)
   DO 1 I=1,N
   DO 1 J=1,M
1  C(I,J)=Q*A(I,J)
   RETURN
   END

```

```

C   SUBROUTINE RECIP(EP,A,X,KER,N)
   THIS SUBROUTINE TAKES THE INVERSE OF A NXN MATRIX
   DIMENSION A(N,N),X(N,N)
   DO 1 I=1,N
   DO 1 J=1,N
1  X(I,J)=0.0
   DO 2 K=1,N
2  X(K,K)=1.0
10 DO 34 L=1,N
   KP=0
   Z=0.0
   DO 12 K=L,N
   IF(Z. GE. ABS(A(K,L))) GO TO 12
11 Z=ABS(A(K,L))
   KP=K
12 CONTINUE
   IF(L. GE. KP) GO TO 20

```



```

13 DO 14 J=L,N
    Z=A(L,J)
    A(L,J)=A(KP,J)
14 A(KP,J)=Z
    DO 15 J=1,N
    Z=X(L,J)
    X(L,J)=X(KP,J)
15 X(KP,J)=Z
20 IF(ABS(A(L,L)).LE.EP) GO TO 50
30 IF(L.GE.N) GO TO 34
31 LP1=L+1
    DO 36 K=LP1,N
    IF(A(K,L).EQ.0.) GO TO 36
32 RATIO=A(K,L)/A(L,L)
    DO 33 J=LP1,N
33 A(K,J)=A(K,J)-RATIO*A(L,J)
    DO 35 J=1,N
35 X(K,J)=X(K,J)-RATIO*X(L,J)
36 CONTINUE
34 CONTINUE
40 DO 43 I=1,N
    II=N+1-I
    DO 43 J=1,N
    S=0.
    IF(II.GE.N) GO TO 43
41 IIP1=II+1
    DO 42 K=IIP1,N
42 S=S+A(II,K)*X(K,J)
43 X(II,J)=(X(II,J)-S)/A(II,II)
    KER=1
    RETURN
50 KER=2
    RETURN
    END

```


C DIODE SOLUTION USING NON LINEAR STATE VARIABLE FORMAT

```

REAL*8 ITITLE
DIMENSION AIPP(905)
DIMENSION V(600),CURR(600) ,XP(905),YP(905),ITITLE(12)
CALL CANCEL(2)
3  FORMAT(8E10.2)
11  FORMAT(6A8)
12  FORMAT(A4)
13  FORMAT(//,1X,'L=',I10)
300  FORMAT(1E10.2,2I10)
550  FORMAT(10X,3E15.5)
699  FORMAT('1',//,7X,'TD',13X,'R',14X,'RADC',11X,'TR',13X,
1'CD')
700  FORMAT(/,5E15.6)
READ(5,12) LABEL1
READ(5,11) ITITLE
DO 991 LOOP= 1,5
READ(5,300) DVMIN,NSTOP,IEND
ICNT=1
100  TIME=0.0
READ(5,3)T,SI,E,R,TR,TD,RADC,CD
WRITE(6,699)
WRITE(6,700) TD,R,RADC,TR,CD
TAU= 1.E-07
TEMT=0.0
ALP=1./0.026
RSH=1.E+08
G=1./R+1./RSH
Z=RADC*(1.-EXP(-TAU/TR))
V(1) = -E
CURR(1)=0.0
KKK=590
KKKIN=KKK-1
NM1=1
N=1
4  N=N+1
T=2.*T
IF(T.GE.1.E-10) T=1.E-10
Y=TIME+T
CON1=SI*ALP* EXP(ALP*V(NM1))
CON2=TD*CON1
CON3=ALP*CCN2
CON4=SI*( EXP(ALP*V(NM1))-1.)
DEN=CD+CON2
DEN2=(DEN**2)
F1=-(V(NM1)*G+CON4)/DEN
B= 1./DEN
PF1=-(G+CON1)/DEN+(CON3*G*V(NM1)+CON3*CON4)/DEN2
PB= -CON3/DEN2
9  IF(Y.LT.TAU) GO TO 1
CURR(N) = Z*EXP((-Y+TAU)/TR)
GO TO 2
1  CURR(N) = RADC*(1.-EXP(-Y/TR))
2  CONTINUE
UNM1=CURR(NM1)-E/R
UN=CURR(N)-E/R
UT=(UN+UNM1)/2.
V(N)=V(NM1)+(T*(F1+B *UT))/(1.-T*(PF1+PB *UN)/2.)
DELV=V(N)-V(NM1)
IF( ABS(DELV).LT.DVMIN) GO TO 7
T=T/2.
Y = TIME + T
GO TO 9
7  GOLD=GNEW
TIME=TIME+T
IF(N.GE.NSTCP) GO TO 20
TEMT=TEMT+T
IF(TEMT.GE..15E-08) GO TO 20
NM1=N

```



```

GO TO 4
20 KKK=KKK+1
   II=KKK-KKKIN
   XP(II)=TIME
   YP(II)=-V(N)
   AIPP(II)=CURR(N)
   WRITE(6,550) XP(II),YP(II),AIPP(II)
   NM1=N
   N=1
   TEMT=0.0
   REC = 0.9*E
   IF(YP(II).LT.0.0) GO TO 55
   IF(DELV.GT.0.0) GO TO 55
   IF(YP(II).GE.REC) GO TO 506
55  CONTINUE
   IF(KKK.LT.1493) GO TO 4
506 CONTINUE
   XP(1)=0.0
   YP(1)=E
   AIPP(1)= 0.0
C   PLOT THE DATA POINTS
   IF(IEND.EQ.1) GO TO 104
   IF(ICNT.GT.1) GO TO 102
   CALL DRAW(II,XP,YP,1,0,LABEL1,ITITLE,3.E-07,3.,1,0,2,0
1,5,5,1,L)
   WRITE(6,13) L
   GO TO 103
102 IF(ICNT.EQ.IEND) GO TO 101
   CALL DRAW(II,XP,YP,2,0,LABEL1,ITITLE,3.E-07,3.,1,0,2,0
1,5,5,1,L)
   WRITE(6,13) L
103 ICNT=ICNT+1
   GO TO 100
101 CALL DRAW(II,XP,YP,3,0,LABEL1,ITITLE,3.E-07,3.,1,0,2,0
1,5,5,1,L)
   WRITE(6,13) L
   GO TO 106
104 CALL DRAW(II,XP,YP,0,0,LABEL1,ITITLE,3.E-07,3.,1,0,2,0
1,5,5,1,L)
   WRITE(6,13) L
106 CONTINUE
991 CONTINUE
   RETURN
   END

```


C SINGLE PRECISION SENSITIVITY ANALYSIS - DIODE PROBLEM

```

REAL*8 ITITLE
DIMENSION ITITLE(12),CURR(600),XP(905),V(600),YP(905),
1XS1(600),S1(905),XS2(600),S2(905),XS3(600),S3(905),
2XS4(600),S4(905),SCUR(600),XS5(600),S5(905),XS6(600),
3S6(905),TCUR(600)
CALL CANCEL(2)
NSTOP=300.
3 FORMAT(8E10.2)
11 FORMAT(6A8)
12 FORMAT(A4)
13 FORMAT(/,1X,'L=',I10)
14 FORMAT(10E12.4,I5)
TIME=0.0
READ(5,12) LABEL1
READ(5,3)T,SI,E,R,TR,TD,RADC,CD
READ(5,3) VMIN,S1MIN,S2MIN,S3MIN,S4MIN,S5MIN,S6MIN
READ(5,3) V(1),XP(1),YP(1),CURR(1),SCUR(1),TCUR(1)
READ(5,3) XS1(1),XS2(1),XS3(1),XS4(1),XS5(1),XS6(1)
READ(5,3) S1(1),S2(1),S3(1),S4(1),S5(1),S6(1)
TEMT=0.0
ALP=1./0.026
RSH=1.E+08
G=1./R+1./RSH
UO=-E/R
US1O=0.
US2O=-E/R
US3O=E/R
US4O=0.
US5O=0.
US6O=0.
US1N=0.
US2N=-E/R
US3N=E/R
US5N=0.
US1T=0.
US2T=US2O+US2N
US3T=US3O+US3N
US5T=0.
GOLD=RADC
KKK=590
KKKIN=KKK-1
NM1=1
N=1
4 IF(TIME.LT.1.E-07) GO TO 1
GNEW=0.0
GO TO 2
1 GNEW=RADC
2 N=N+1
T=2.*T
IF(T.GE.1.E-10) T=1.E-10
CON1=SI*ALP*EXP(ALP*V(NM1))
CON2=TD*CON1
CCN3=ALP*CCN2
CON4=SI*(EXP(ALP*V(NM1))-1.)
DEN=CD+CON2
DEN2=DEN**2
DEN3=DEN**3
F=- (G*V(NM1)+CON4)/DEN
B=1./DEN
PF=- (G+CON1)/DEN+(CON3*(G*V(NM1)+CON4))/DEN2
PB=-CON3/DEN2
FS1=-((G+CON1)*XS1(NM1))/DEN+((CON4+G*V(NM1))*(CON2
1+CON3*XS1(NM1)))/DEN2
FS2=-((G+CCN1)*XS2(NM1))/DEN+(CON3*XS2(NM1)*(G*V(NM1)+
1CON4))/DEN2
FS3=-((G+CON1)*XS3(NM1)-V(NM1)/R)/DEN+(CON3*XS3(NM1)*
1(G*V(NM1)+CCN4))/DEN2
FS4=-((G+CON1)*XS4(NM1))/DEN+(CON3*XS4(NM1)*(G*V(NM1)+
1CON4))/DEN2

```



```

FS5=-((G+CON1)*XS5(NM1))/DEN+((G*V(NM1)+CON4)*(CD+CON3
1*XS5(NM1)))/DEN2
FS6=-((G+CON1)*XS6(NM1))/DEN+(CON3*XS6(NM1)*(G*V(NM1)+
1CON4))/DEN2
BS1=-((CON2+CON3*XS1(NM1))/DEN2
BS2=-((CON3*XS2(NM1))/DEN2
BS3=-((CON3*XS3(NM1))/DEN2
BS4=-((CON3*XS4(NM1))/DEN2
BS5=-((CD+CON3*XS5(NM1))/DEN2
BS6=-((CON3*XS6(NM1))/DEN2
PFS1=-((G+CON1)/DEN+(CON3*(CON4+G*V(NM1)))/DEN2
PFS2=PFS1
PFS3=PFS1
PFS4=PFS1
PFS5=PFS1
PFS6=PFS1
PXFS1=-((ALP*CON1*XS1(NM1))/DEN+((G+CON1)*(CON2+2.*CON3
1*XS1(NM1)))+(G*V(NM1)+CON4)*(CON3+ALP*CON3*XS1(NM1)))/
2DEN2-(2.*CON3*(G*V(NM1)+CON4)*(CON2+CON3*XS1(NM1)))/
3DEN3
PXFS2=-((ALP*CON1*XS2(NM1))/DEN+(ALP*CON3*XS2(NM1)*(G*
1V(NM1)+CON4)+2.*CON3*XS2(NM1)*(G+CON1))/DEN2-(2.*CON3*
2CON3*XS2(NM1)*(G*V(NM1)+CON4))/DEN3
PXFS3=-((ALP*CON1*XS3(NM1)-1./R)/DEN+(ALP*CON3*XS3(NM1)
1*(G*V(NM1)+CON4)+2.*CON3*XS3(NM1)*(G+CON1)-CON3*
2V(NM1)/R)/DEN2-(2.*CON3*CON3*(G*V(NM1)+CON4)*XS3(NM1))
3/DEN3
PXFS4=-((ALP*CON1*XS4(NM1))/DEN+(ALP*CON3*XS4(NM1)*(G*
1V(NM1)+CON4)+2.*CON3*XS4(NM1)*(G+CON1))/DEN2-(2.*CON3*
2CON3*XS4(NM1)*(G*V(NM1)+CON4))/DEN3
PXFS5=-((ALP*CON1*XS5(NM1))/DEN+(G+CON1)*(CD+2.*CON3*
1XS5(NM1)))+(G*V(NM1)+CON4)*(ALP*CON3*XS5(NM1))/DEN2-(
22.*CON3*(G*V(NM1)+CON4)*(CD+CON3*XS5(NM1)))/DEN3
PXFS6=-((ALP*CON1*XS6(NM1))/DEN+(ALP*CON3*XS6(NM1)*(G*
1V(NM1)+CON4)+2.*CON3*XS6(NM1)*(G+CON1))/DEN2-(2.*CON3*
2CON3*XS6(NM1)*(G*V(NM1)+CON4))/DEN3
PBS1=-CON3/DEN2
PBS2=PBS1
PBS3=PBS1
PBS4=PBS1
PBS5=PBS1
PBS6=PBS1
PXBS1=-((CON3*(1.+ALP*XS1(NM1)))/DEN2+(2.*CON3*(CON2+
1CON3*XS1(NM1)))/DEN3
PXBS2=-((ALP*CON3*XS2(NM1))/DEN2+(2.*CON3*CON3*XS2(NM1)
1)/DEN3
PXBS3=-((ALP*CON3*XS3(NM1))/DEN2+(2.*CON3*CON3*XS3(NM1)
1)/DEN3
PXBS4=-((ALP*CON3*XS4(NM1))/DEN2+(2.*CON3*CON3*XS4(NM1)
1)/DEN3
PXBS5=-((ALP*CON3*XS5(NM1))/DEN2+(2.*CON3*(CD+CON3*
1XS5(NM1)))/DEN3
PXBS6=-((ALP*CON3*XS6(NM1))/DEN2+(2.*CON3*CON3*XS6(NM1)
1)/DEN3
9 CURR(N)=((2.*TR-T)/(2.*TR+T))*CURR(NM1)+(T/(2.*TR+T))*
1(GOLD+GNEW)
SCUR(N)=((2.*TR-T)/(2.*TR+T))*SCUR(NM1)+(T/(2.*TR+T))*
1(GOLD+GNEW)
TCUR(N)=((2.*TR-T)*TCUR(NM1)+T*(CURR(N)+CURR(NM1)
1-GOLD-GNEW))/(2.*TR+T)
UN=CURR(N)-E/R
US4N=SCUR(N)
US6N=TCUR(N)
UT=UN+UO
US4T=US4N+US4O
US6T=US6O+US6N
V(N)=V(NM1)+(T*(F+.5*B*UT))/(1.-T*(PF+PB*UN)/2.)
DELV=V(N)-V(NM1)
VNORM=ABS(DELV)
IF(VNORM.GT.VMIN) GO TO 5
XS1(N)=XS1(NM1)+(T*(FS1+.5*(DELV*(PXFS1+PB*US1N+PXBS1*
1UN)+B*US1T+BS1*UT)))/(1.-T*(PFS1+PBS1*UN)/2.)

```



```

DELS1=XS1(N)-XS1(NM1)
S1NORM= SQRT(DELV**2+DELS1**2)
IF(S1NORM.GT.S1MIN) GO TO 5
XS2(N)=XS2(NM1)+(T*(FS2+.5*(DELV*(PXFS2+PB*US2N+PXBS2*
1UN)+B*US2T+BS2*UT)))/(1.-(T*(PFS2+PBS2*UN))/2.)
DELS2=XS2(N)-XS2(NM1)
S2NORM= SQRT(DELV**2+DELS2**2)
IF(S2NORM.GT.S2MIN) GO TO 5
XS3(N)=XS3(NM1)+(T*(FS3+.5*(DELV*(PXFS3+PB*US3N+PXBS3*
1UN)+B*US3T+BS3*UT)))/(1.-(T*(PFS3+PBS3*UN))/2.)
DELS3=XS3(N)-XS3(NM1)
S3NORM= SQRT(DELV**2+DELS3**2)
IF(S3NORM.GT.S3MIN) GO TO 5
XS4(N)=XS4(NM1)+(T*(FS4+.5*(DELV*(PXFS4+PB*US4N+PXBS4*
1UN)+B*US4T+BS4*UT)))/(1.-(T*(PFS4+PBS4*UN))/2.)
DELS4=XS4(N)-XS4(NM1)
S4NORM= SQRT(DELV**2+DELS4**2)
IF(S4NORM.GT.S4MIN) GO TO 5
XS5(N)=XS5(NM1)+(T*(FS5+.5*(DELV*(PXFS5+PB*US5N+PXBS5*
1UN)+B*US5T+BS5*UT)))/(1.-(T*(PFS5+PBS5*UN))/2.)
DELS5=XS5(N)-XS5(NM1)
S5NORM= SQRT(DELV**2+DELS5**2)
IF(S5NORM.GT.S5MIN) GO TO 5
XS6(N)=XS6(NM1)+(T*(FS6+.5*(DELV*(PXFS6+PB*US6N+PXBS6*
1UN)+B*US6T+BS6*UT)))/(1.-(T*(PFS6+PBS6*UN))/2.)
DELS6=XS6(N)-XS6(NM1)
S6NORM= SQRT(DELV**2+DELS6**2)
IF(S6NORM.GT.S6MIN) GO TO 5
GO TO 7
5 T=T/2.
GO TO 9
7 GOLD=GNEW
TIME=TIME+T
UO=UN
US4O=US4N
US6O=US6N
IF(N.GE.NSTOP) GO TO 20
TEMT=TEMT+T
IF(TEMT.GE..15E-08) GO TO 20
NM1=N
GO TO 4
20 KKK=KKK+1
II=KKK-KKKIN
XP(II)=TIME
YP(II)=-V(N)
S1(II)=XS1(N)
S2(II)=XS2(N)
S3(II)=XS3(N)
S4(II)=XS4(N)
S5(II)=XS5(N)
S6(II)=XS6(N)
WRITE(6,14) TIME,CURR(N),V(N),XS1(N),XS2(N),XS3(N),
1XS4(N),XS5(N),XS6(N),T,N
NM1=N
N=1
TEMT=0.0
IF(KKK.LT.1493) GO TO 4
C PLOT THE DATA POINTS
READ(5,11) ITITLE
CALL DRAW(900,XP,YP,0,0,LABEL1,ITITLE,3.E-07,5.,0,0,2,
12,5,5,1,L)
WRITE(6,13) L
C PLOT THE SENSITIVITY FUNCTIONS
READ(5,11) ITITLE
CALL DRAW(900,XP,S1,0,0,LABEL1,ITITLE,3.E-07,5.,0,0,2,
12,5,5,1,L)
WRITE(6,13) L
READ(5,11) ITITLE
CALL DRAW(900,XP,S2,0,0,LABEL1,ITITLE,3.E-07,5.,2,0,2,
12,5,5,1,L)
WRITE(6,13) L
READ(5,11) ITITLE

```



```
CALL DRAW(900,XP,S3,0,0,LABEL1,ITITLE,3.E-07,5.,0,0,2,  
12,5,5,1,L)  
WRITE(6,13) L  
READ(5,11) ITITLE  
CALL DRAW(900,XP,S4,0,0,LABEL1,ITITLE,3.E-07,5.,0,0,2,  
12,5,5,1,L)  
WRITE(6,13) L  
READ(5,11) ITITLE  
CALL DRAW(900,XP,S5,0,0,LABEL1,ITITLE,3.E-07,5.,2,0,2,  
12,5,5,1,L)  
WRITE(6,13) L  
READ(5,11) ITITLE  
CALL DRAW(900,XP,S6,0,0,LABEL1,ITITLE,3.E-07,5.,2,0,2,  
12,5,5,1,L)  
WRITE(6,13) L  
106 RETURN  
END
```

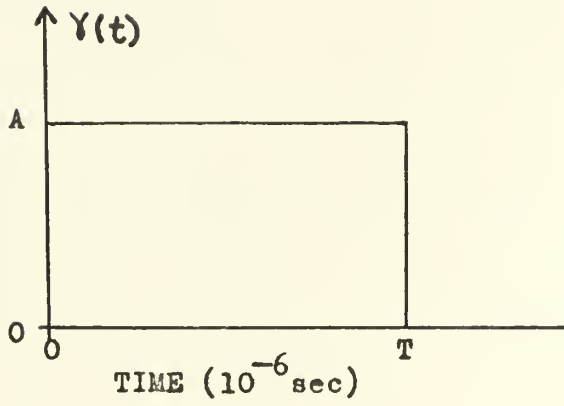



FIG. 2.1 RADIATION CURRENT PULSE

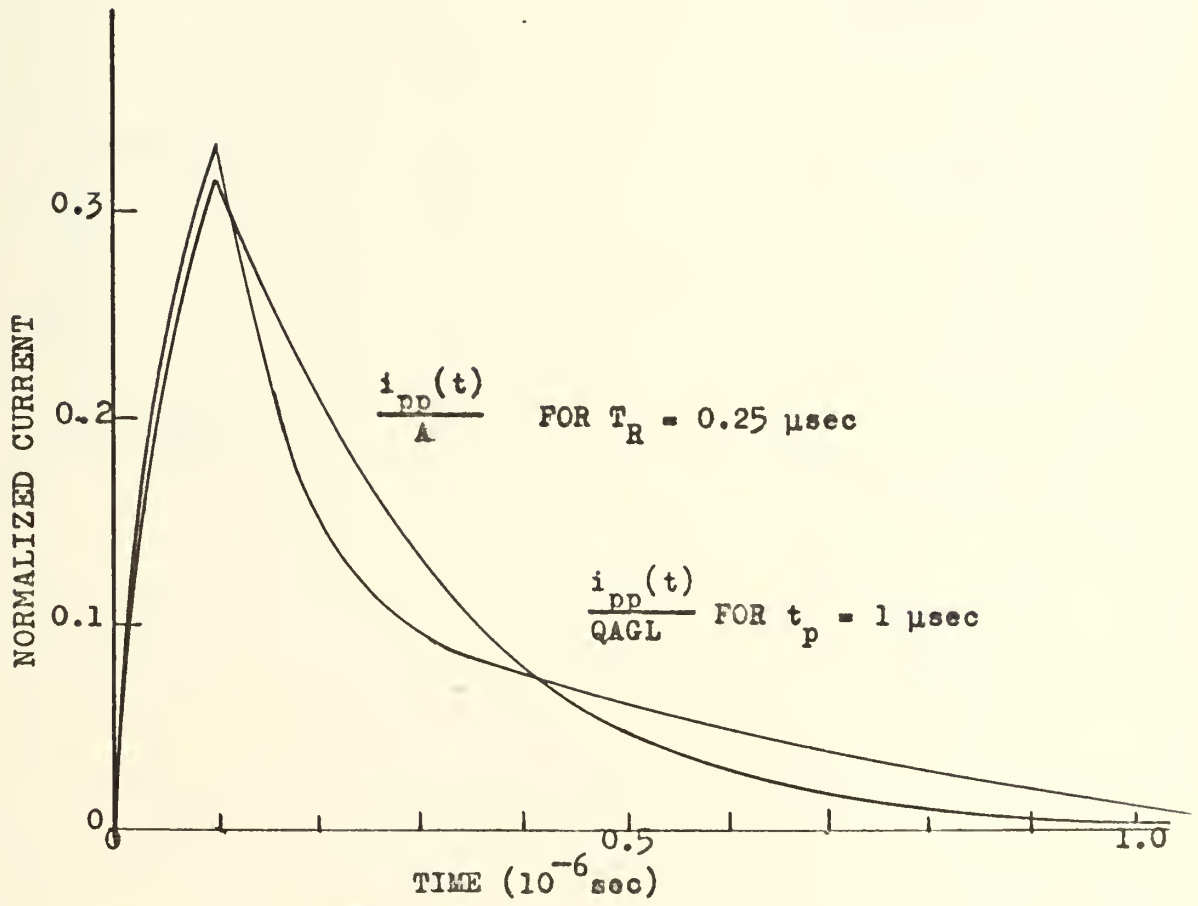


FIG. 2.2 PHOTOCURRENT COMPARISONS

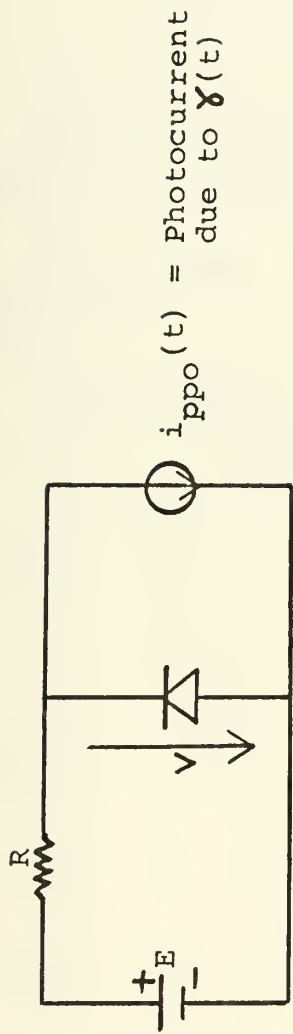


FIG. 3.1 DIODE CIRCUIT

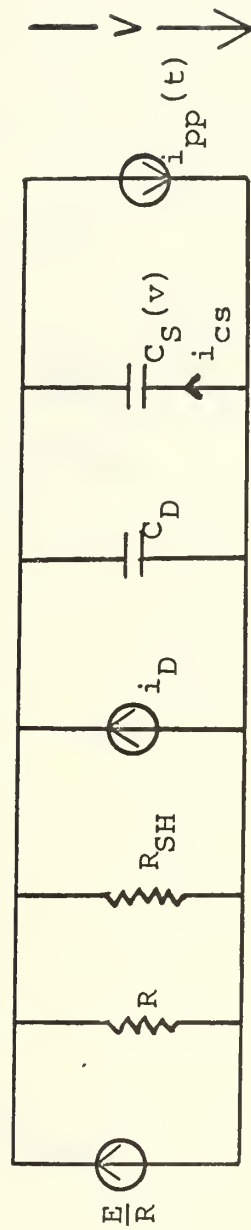


FIG. 3.2 EQUIVALENT CIRCUIT

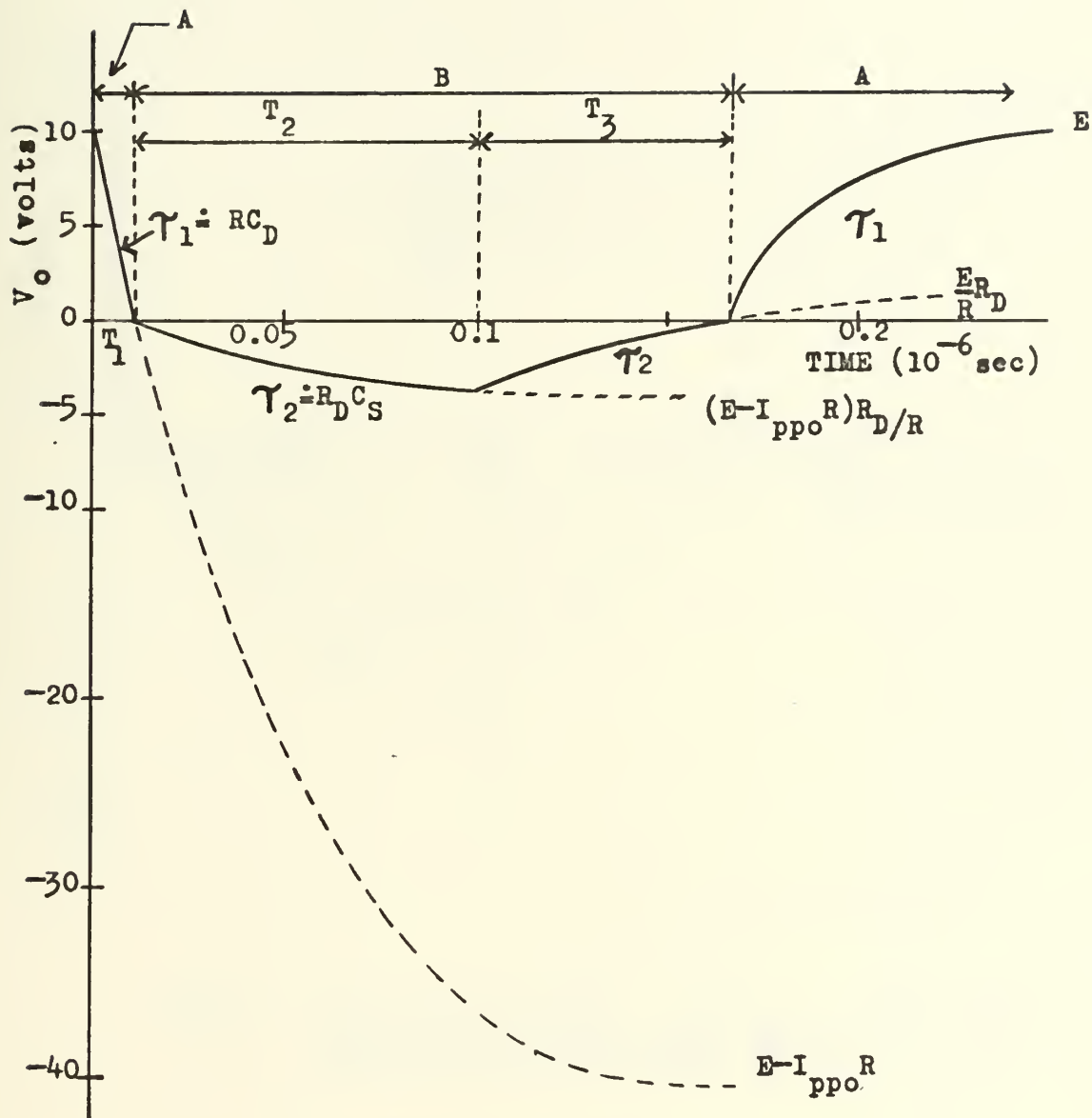


FIG. 3.3 JUNCTION RESPONSE. PIECEWISE LINEAR MODEL. RECTANGULAR CURRENT PULSE.

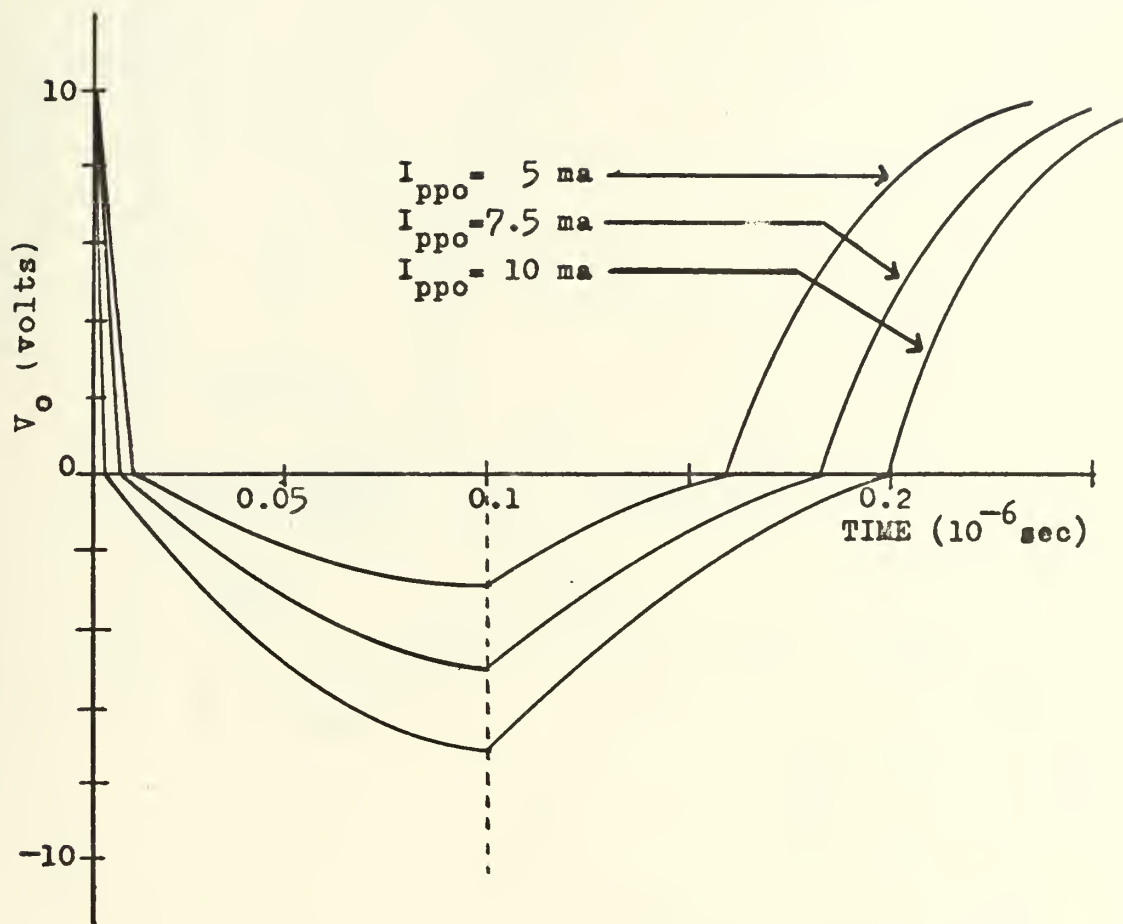


FIG. 3.4 JUNCTION RESPONSE. PIECEWISE LINEAR MODEL. RECTANGULAR PHOTOCURRENT PULSE.

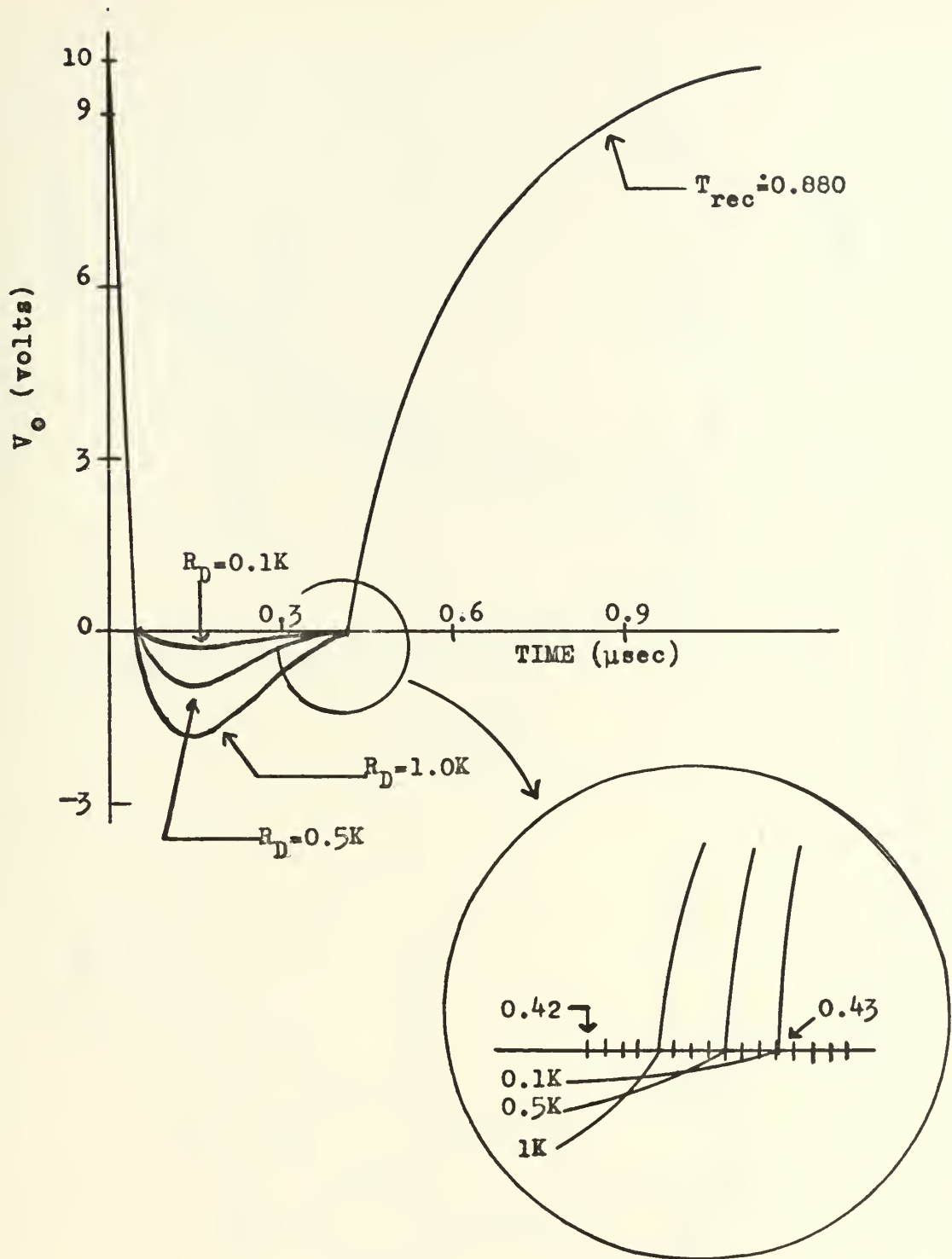


FIG. 3.5 JUNCTION RESPONSE. PIECEWISE LINEAR MODEL. RECTANGULAR RADIATION PULSE.

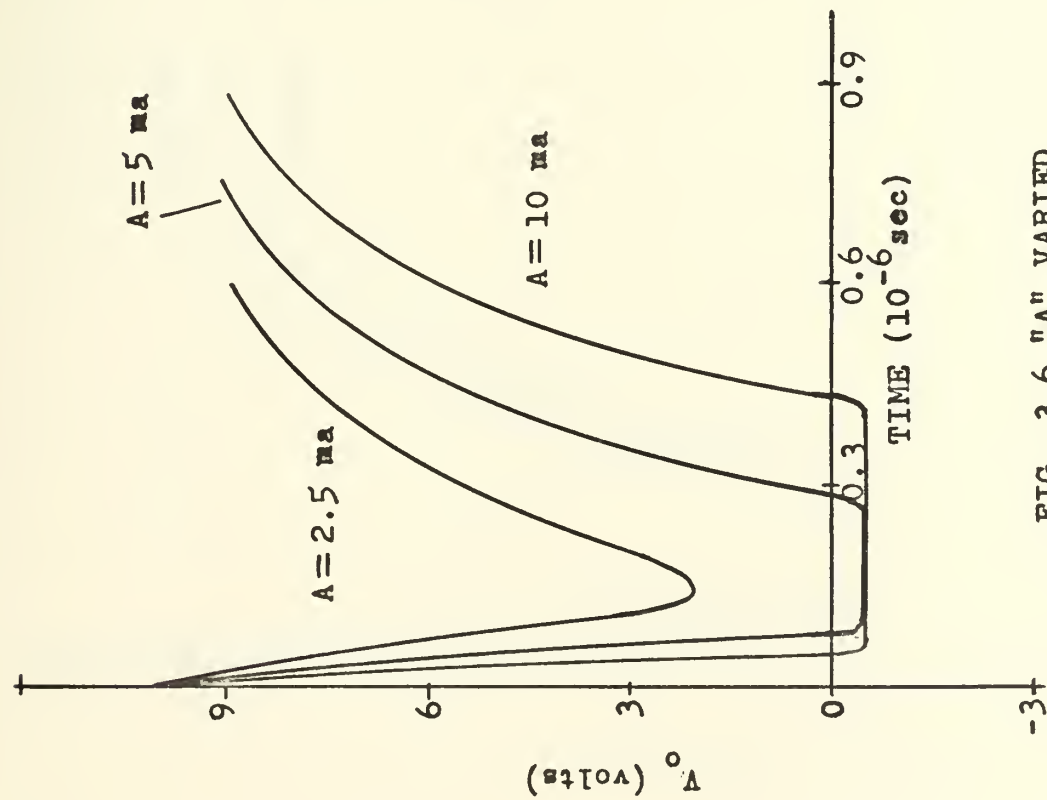


FIG. 3.6 "A" VARIED

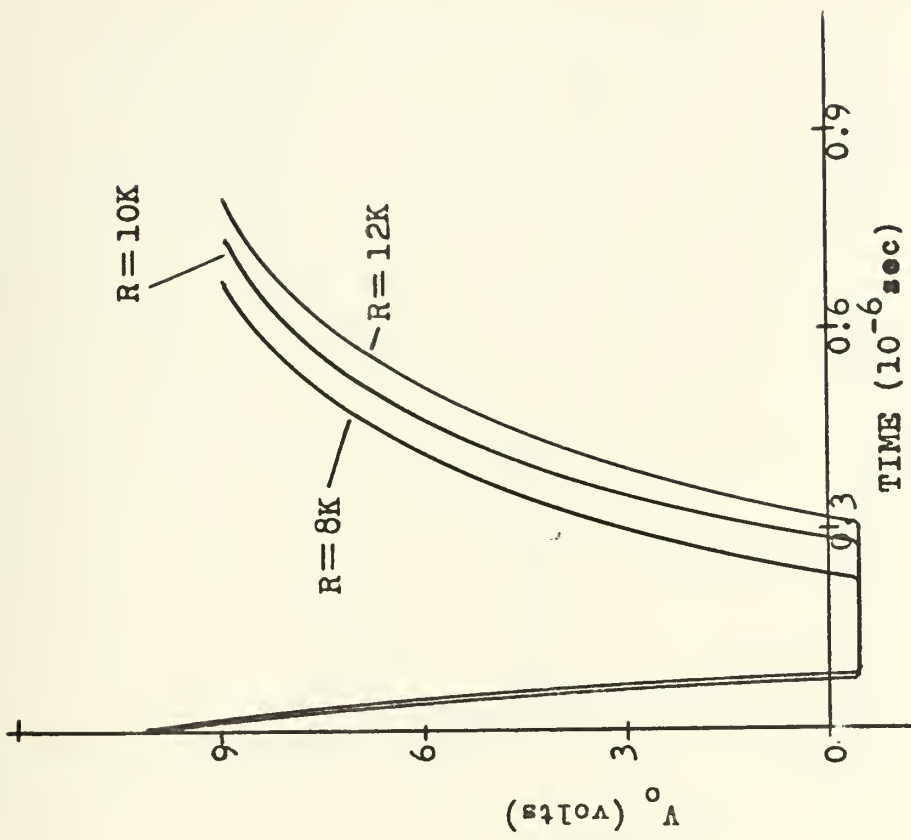


FIG. 3.7 "R" VARIED

OUTPUT VOLTAGE WAVEFORMS

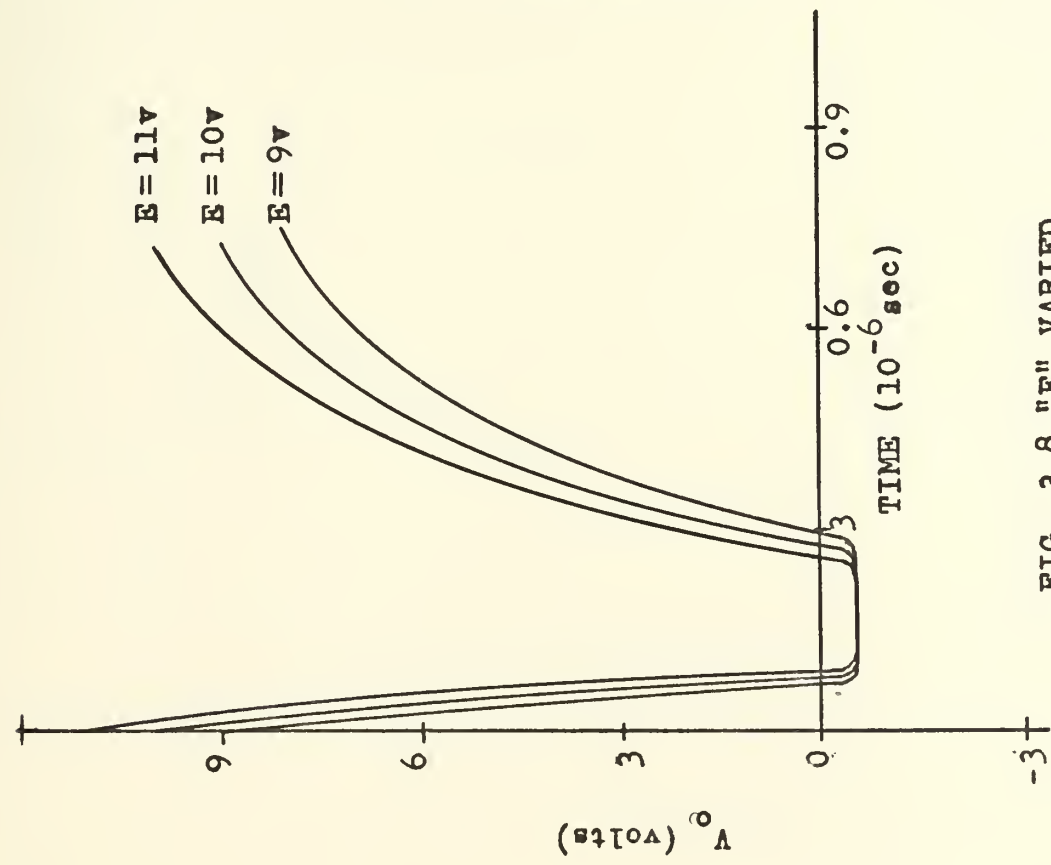


FIG. 3.8 "E" VARIED

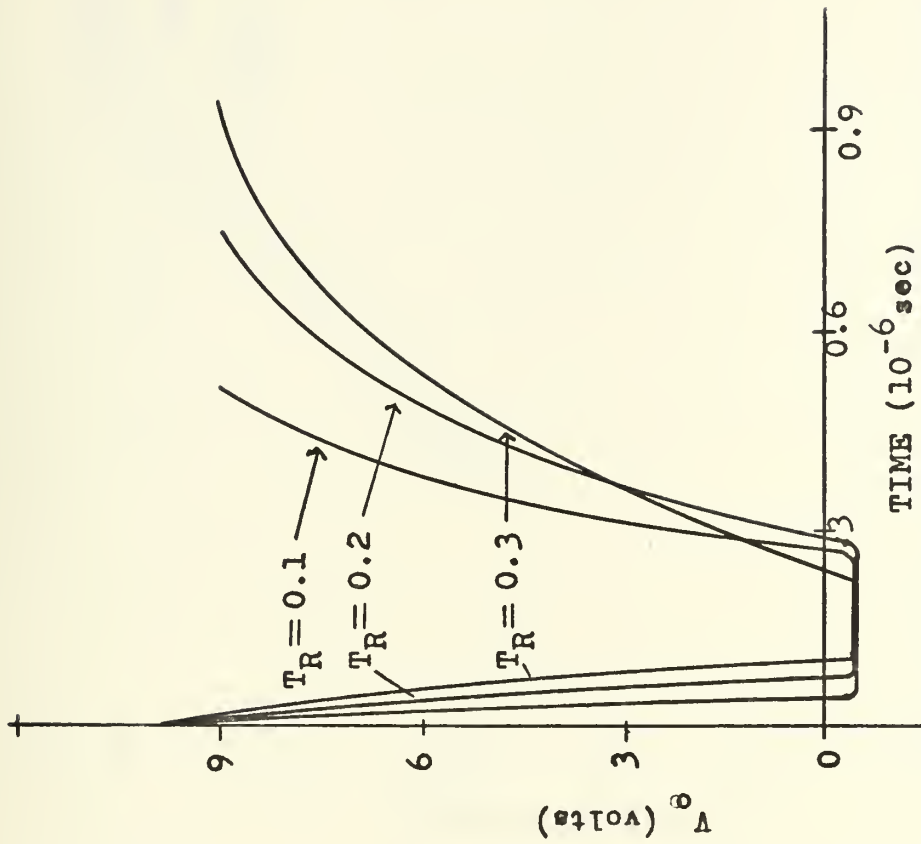


FIG. 3.9 "T_R" VARIED

OUTPUT VOLTAGE WAVEFORMS

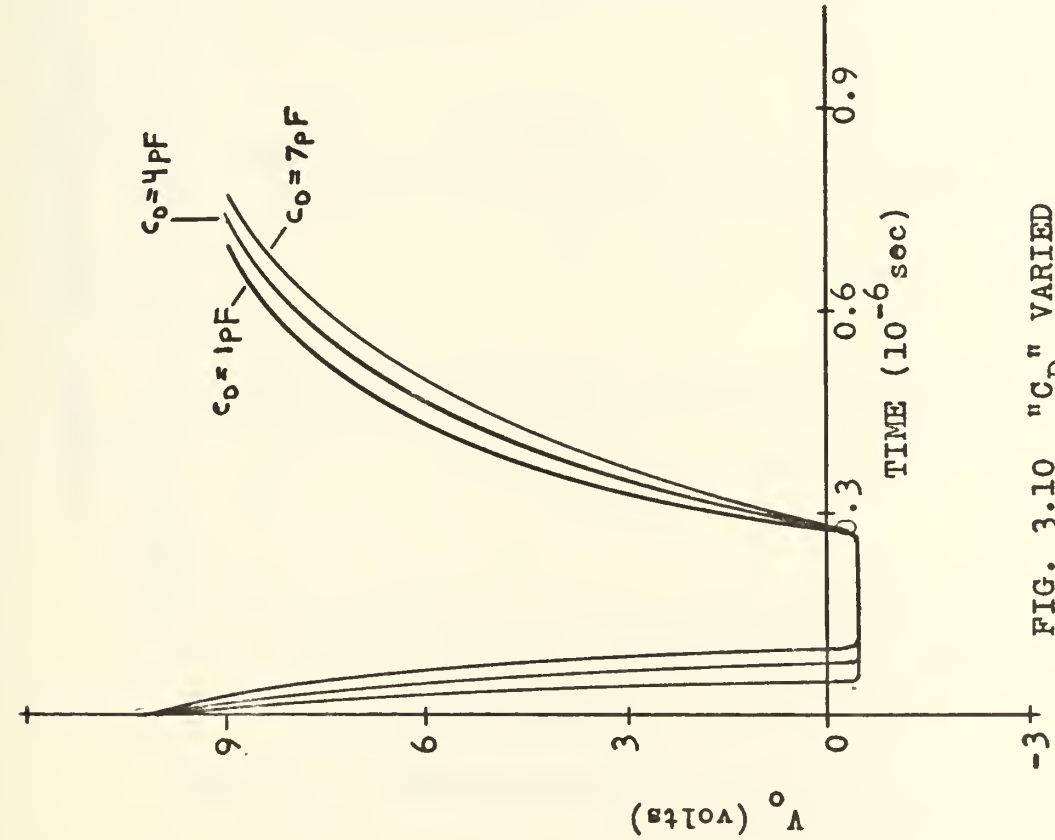


FIG. 3.10 " C_D " VARIED

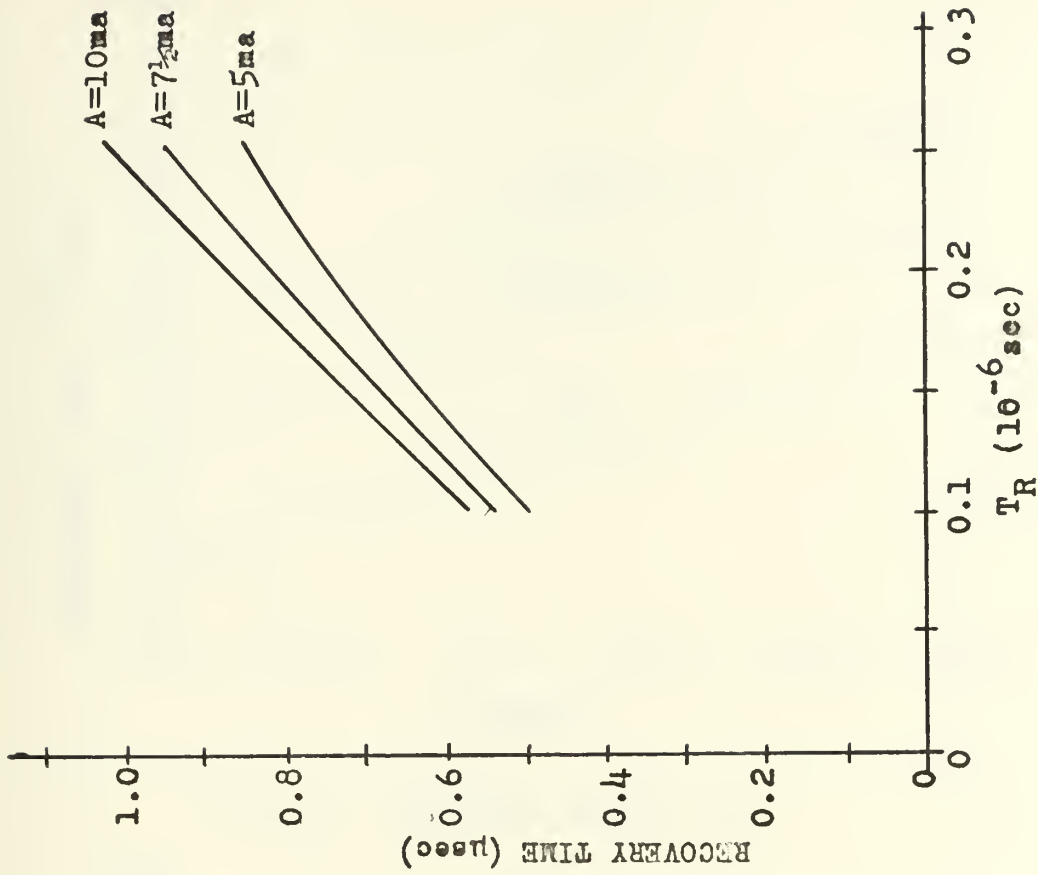


FIG. 3.11 RECOVERY TIME VS. T_R

OUTPUT VOLTAGE WAVEFORMS

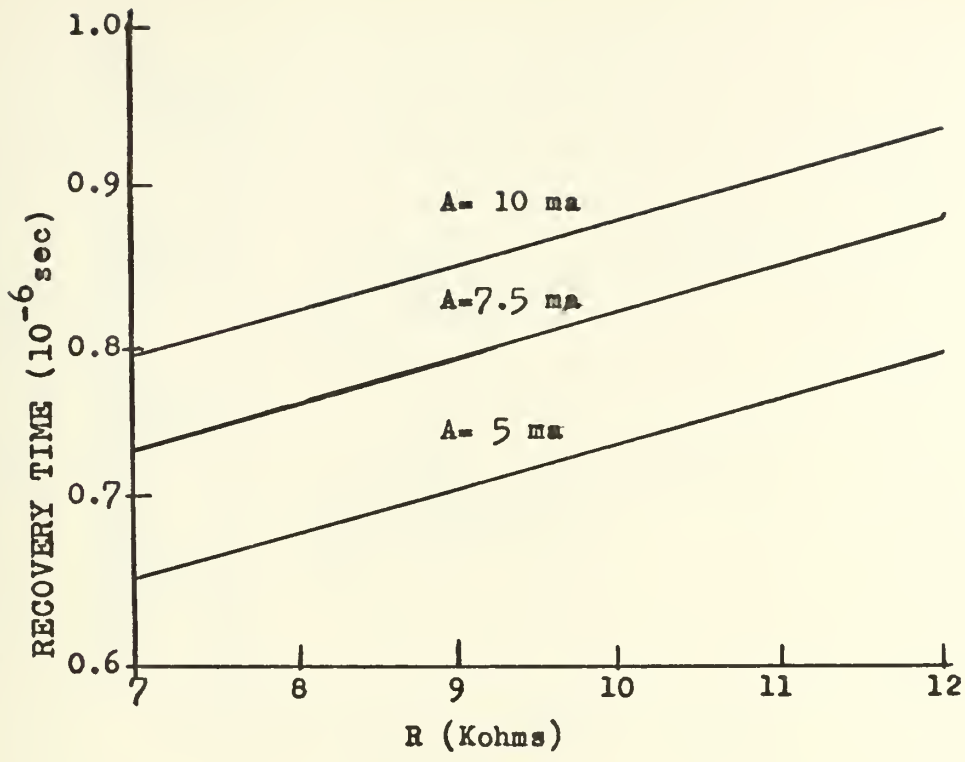


FIG. 3.12 RECOVERY TIME VS. R

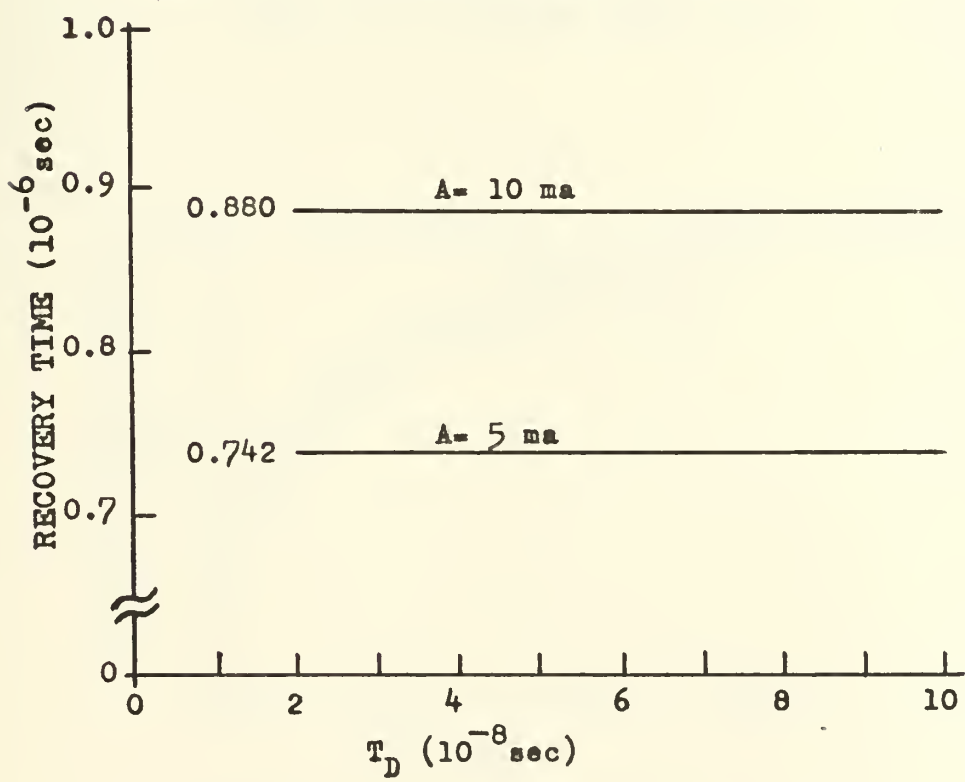


FIG. 3.13 RECOVERY TIME VS. T_D

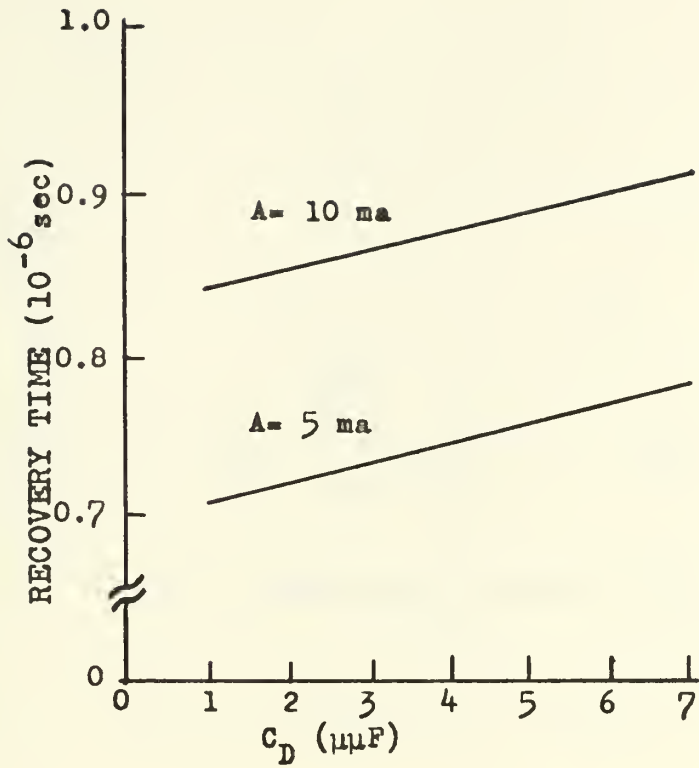


FIG. 3.14 RECOVERY TIME VS. C_D

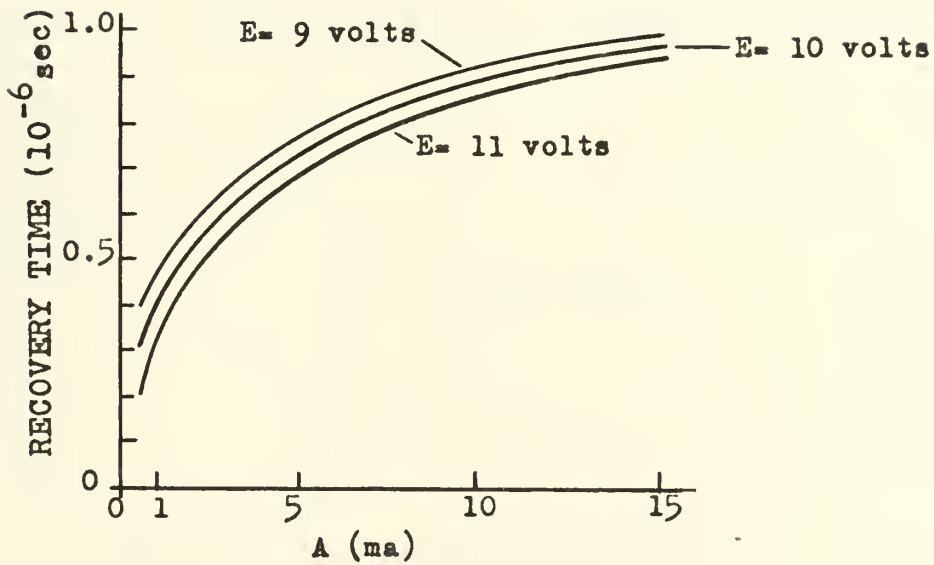


FIG. 3.15 RECOVERY TIME VS. A

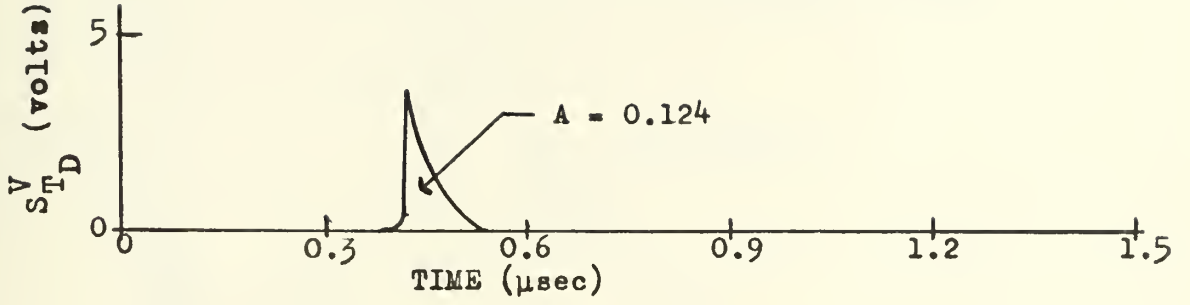


FIG. 4.1 SENSITIVITY FUNCTION S_{TD}^V

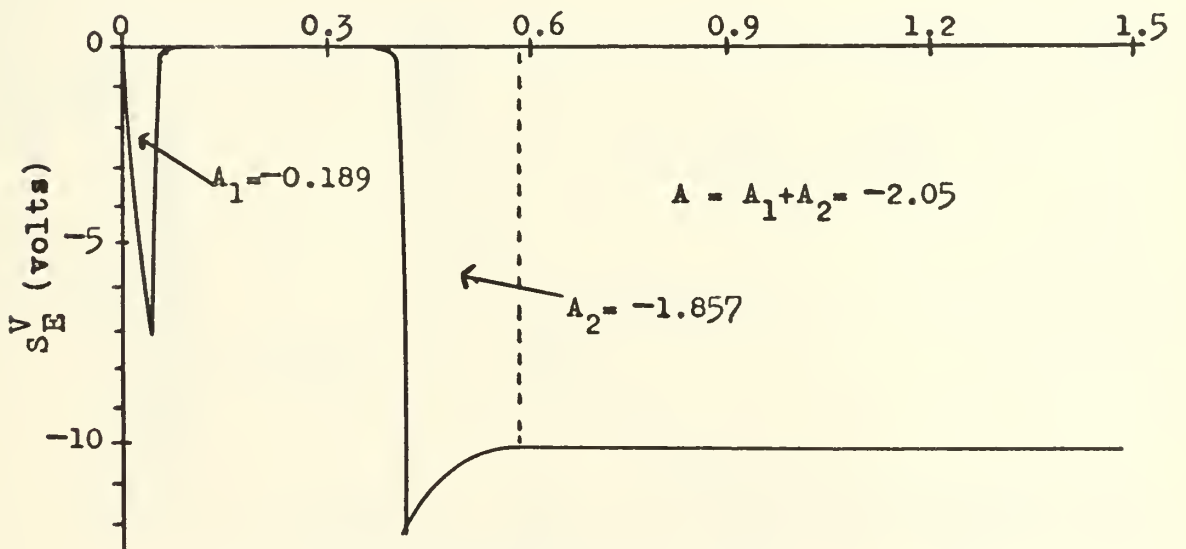


FIG. 4.2 SENSITIVITY FUNCTION S_E^V

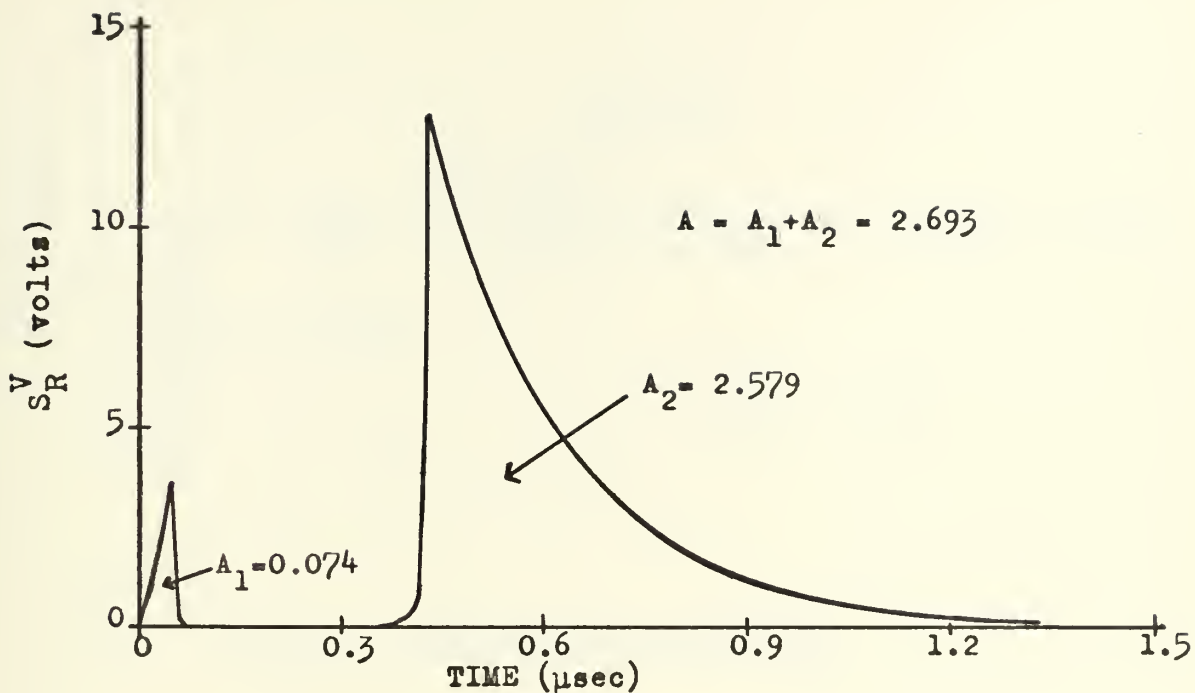


FIG. 4.3 SENSITIVITY FUNCTION S_R^V

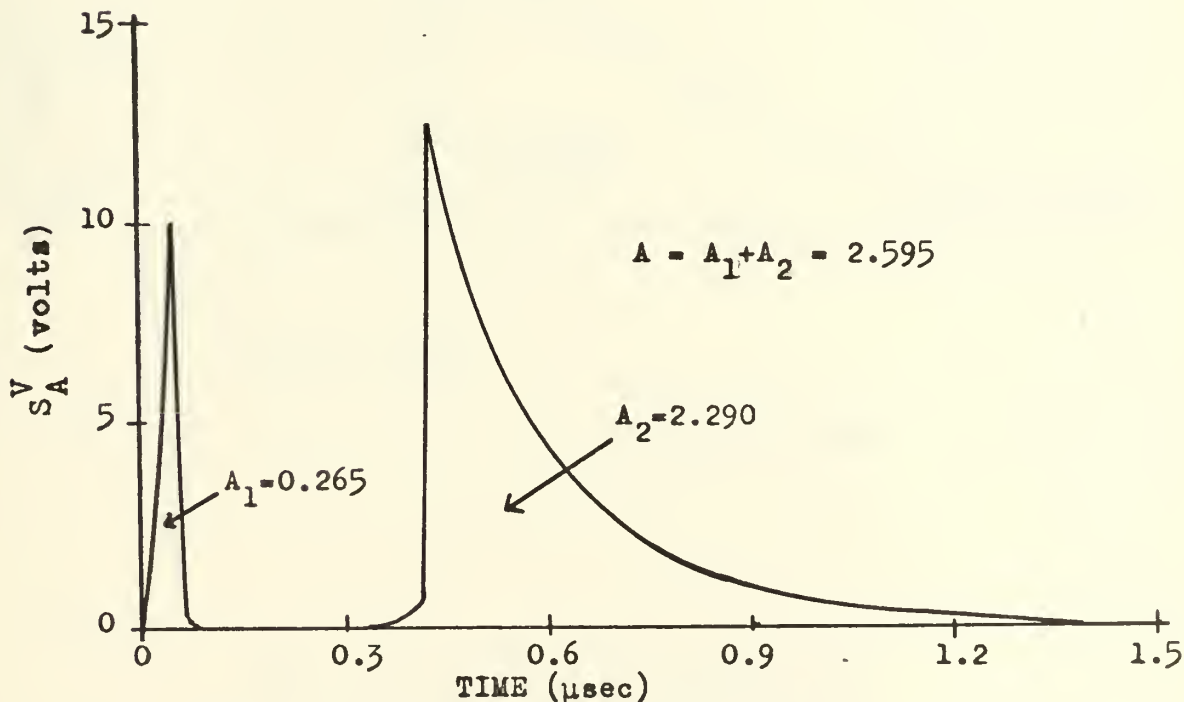


FIG. 4.4 SENSITIVITY FUNCTION S_A^V

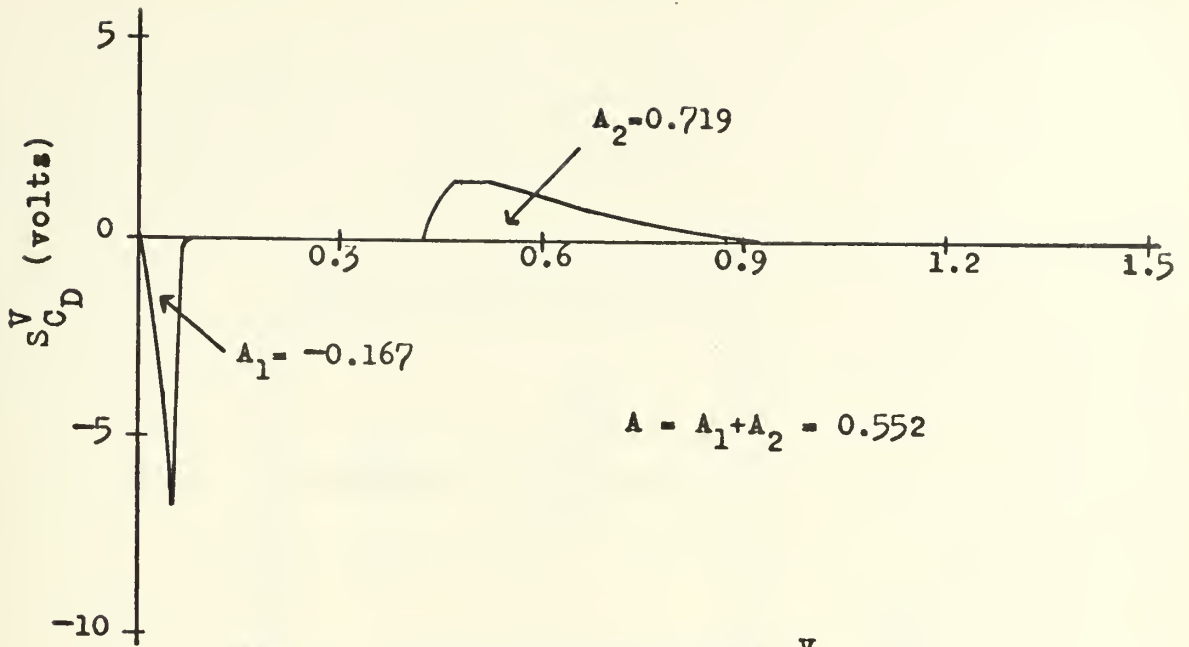


FIG. 4.5 SENSITIVITY FUNCTION S_{CD}^V

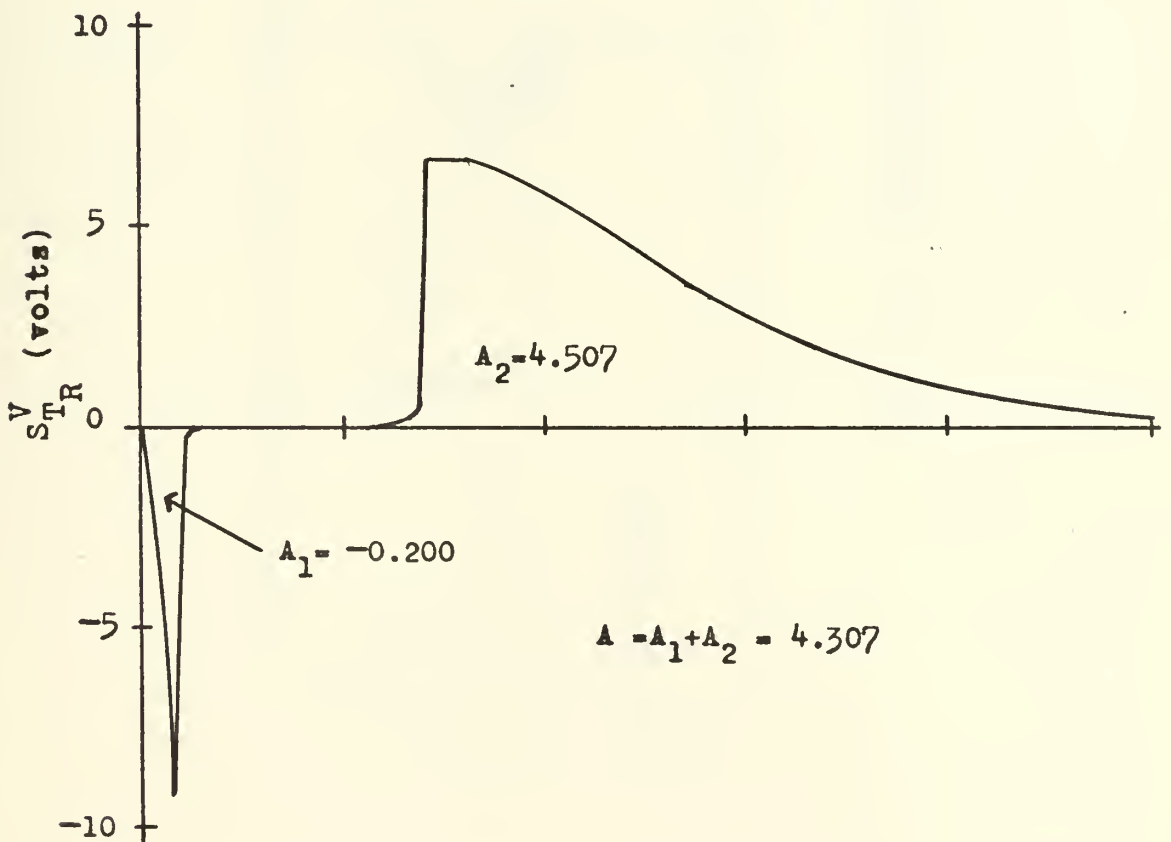


FIG. 4.6 SENSITIVITY FUNCTION S_{TR}^V

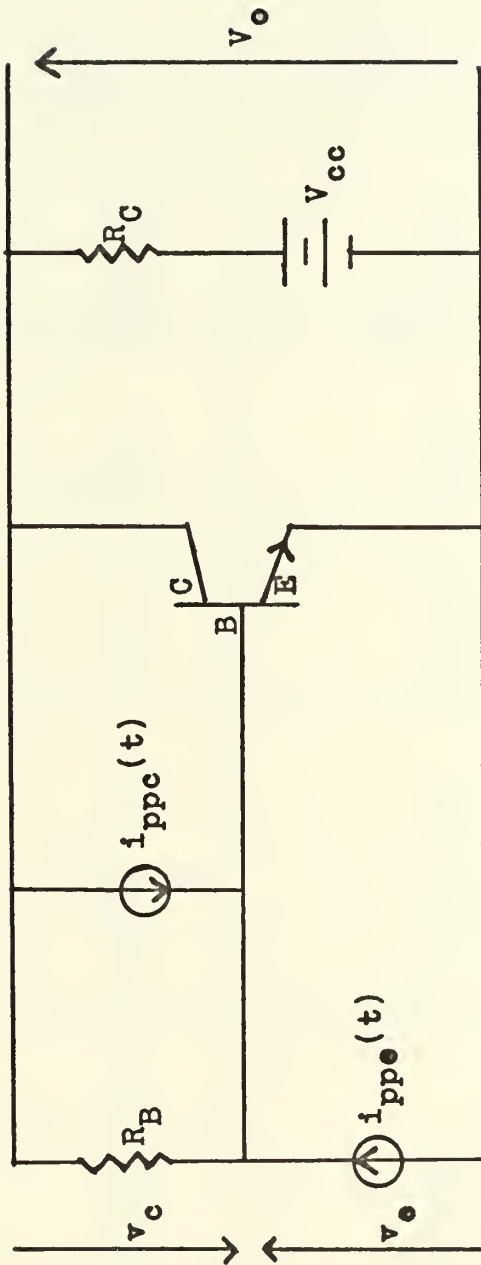


FIG. 5.1 TRANSISTOR CIRCUIT

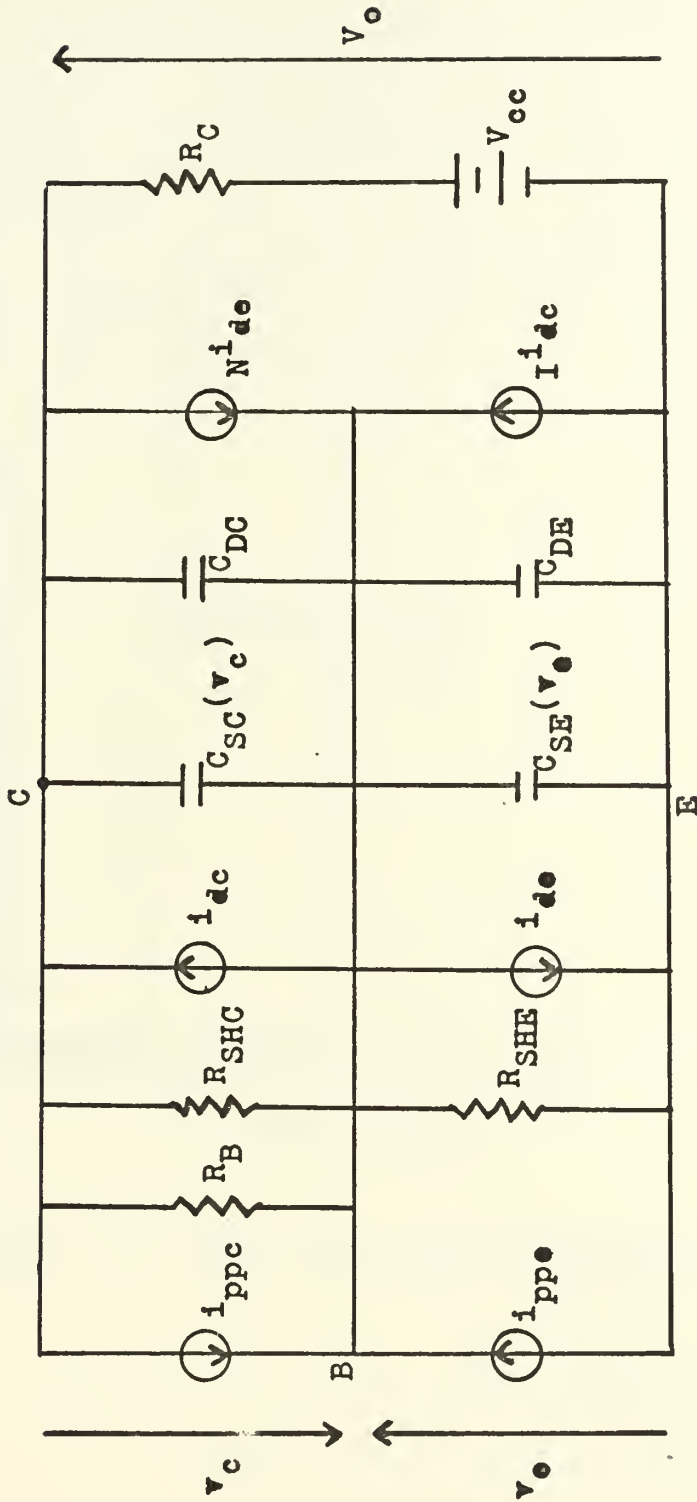


FIG. 5.2 EQUIVALENT TRANSISTOR CIRCUIT

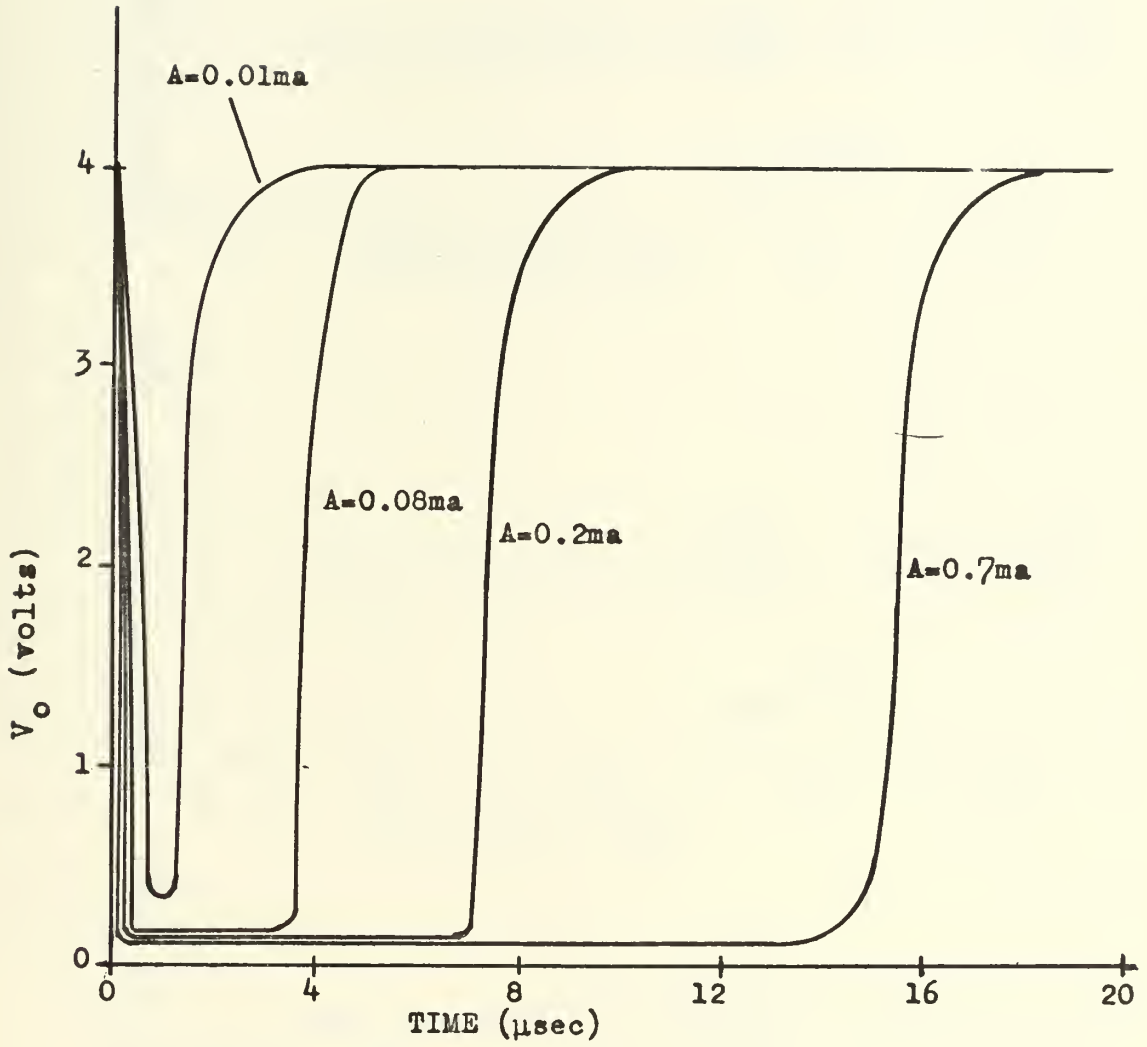


FIG. 5.3 TYPICAL TRANSISTOR VOLTAGE RESPONSE

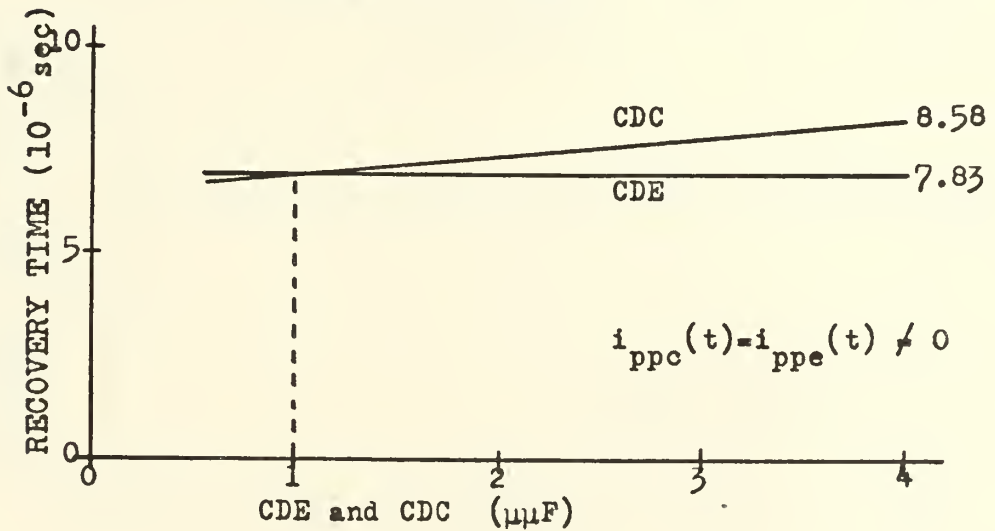
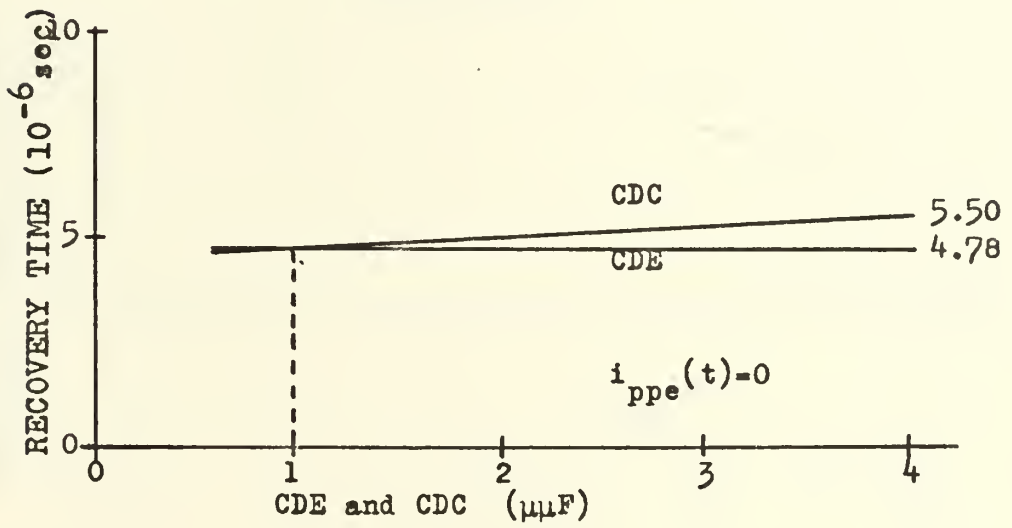
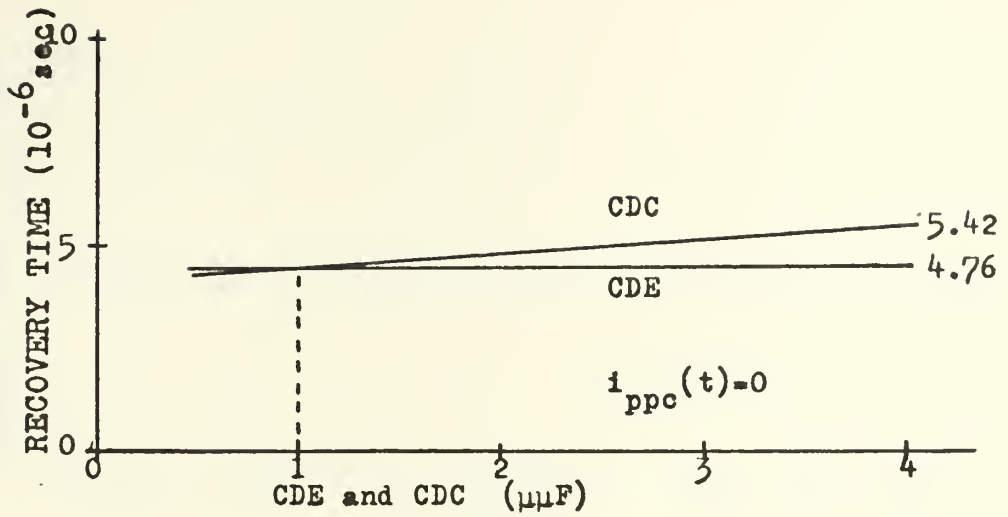


FIG. 5.4 RECOVERY TIME VS. CDE and CDC

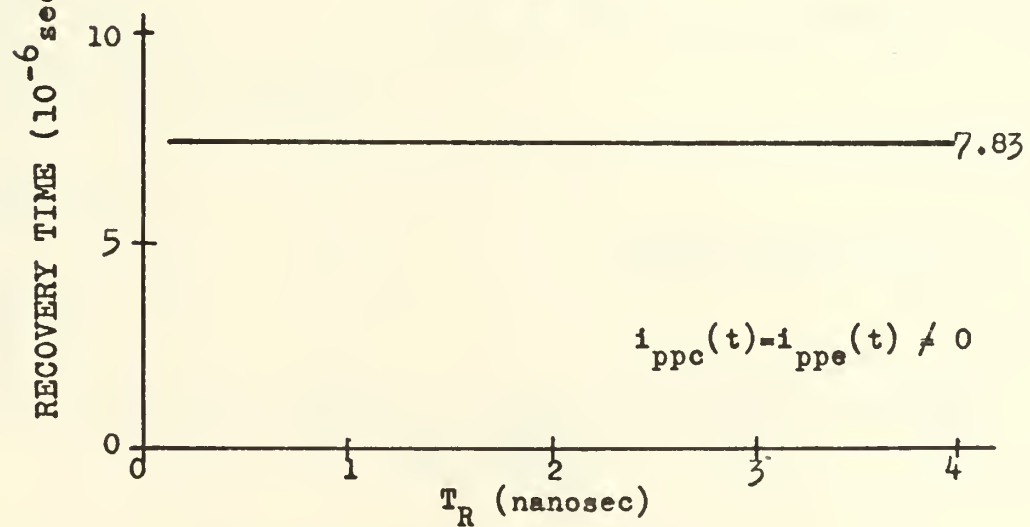
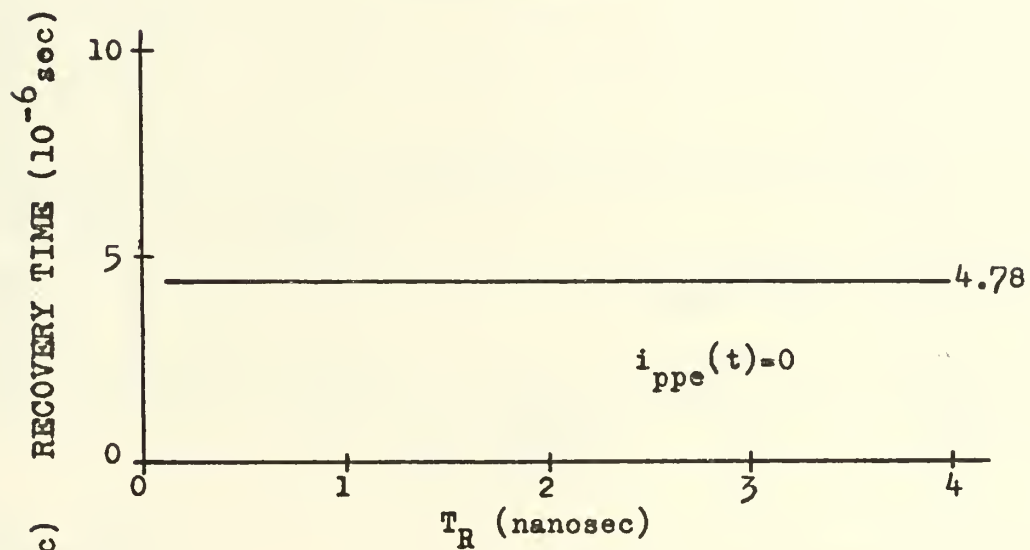
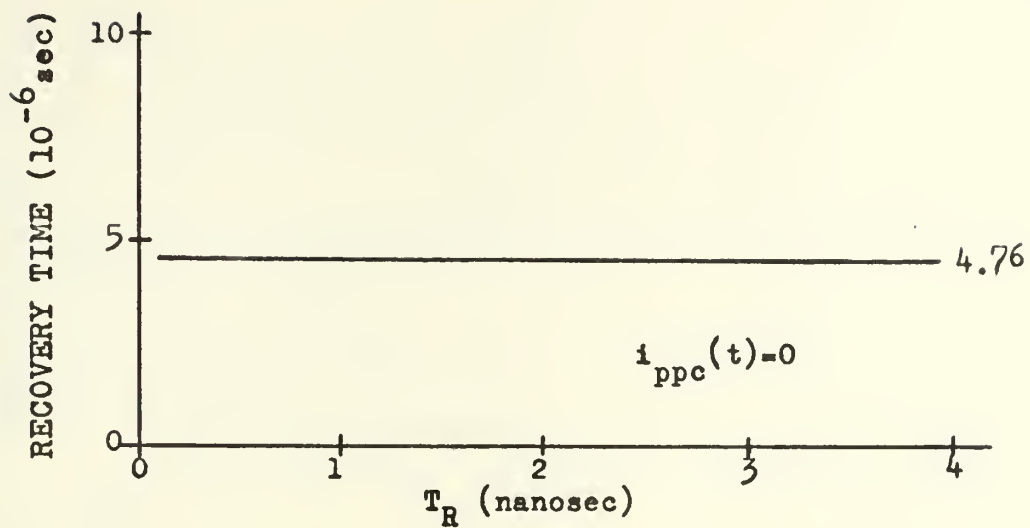


FIG. 5.5 RECOVERY TIME VS. T_R

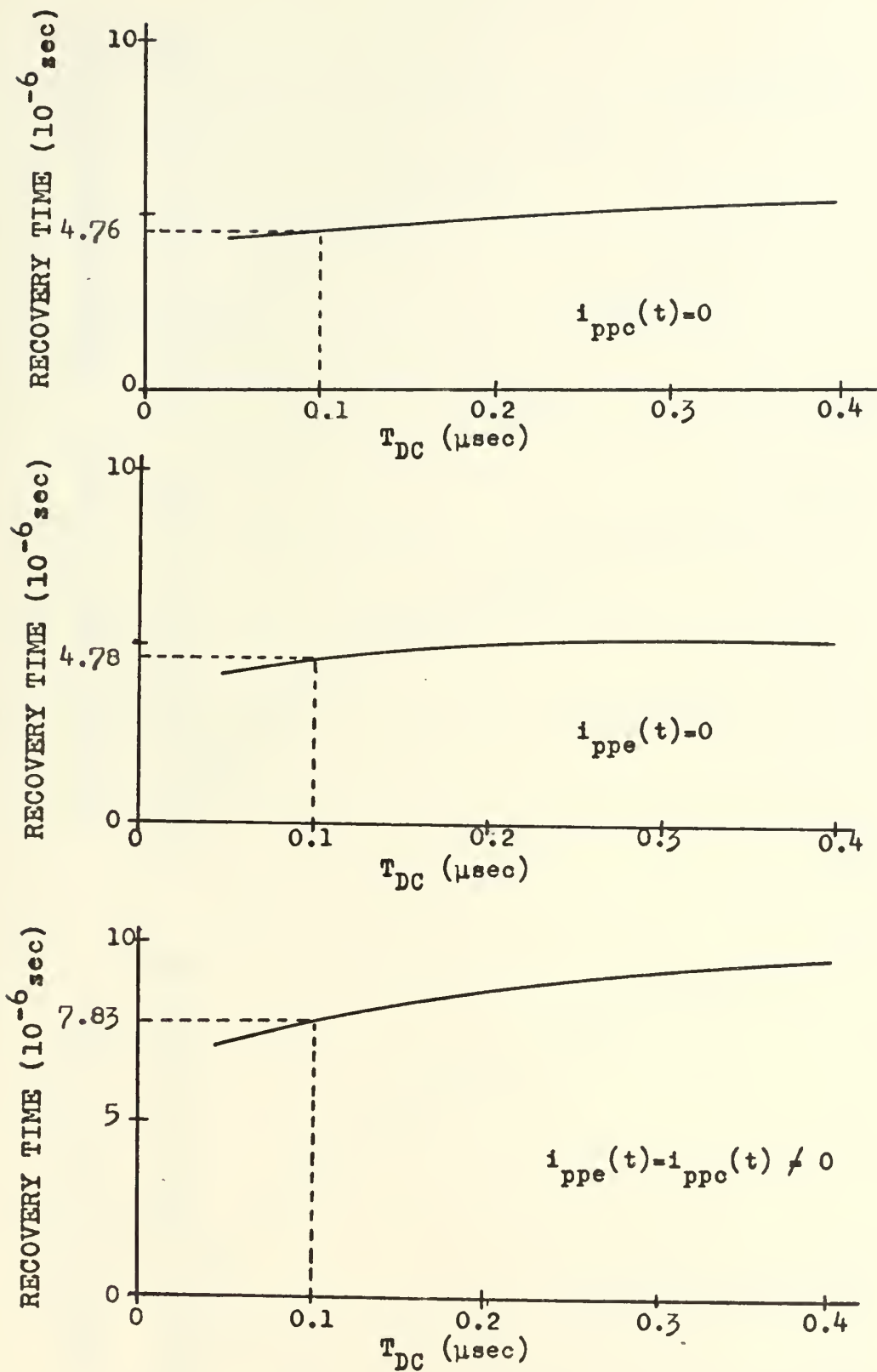


FIG. 5.6 RECOVERY TIME VS. T_{DC}

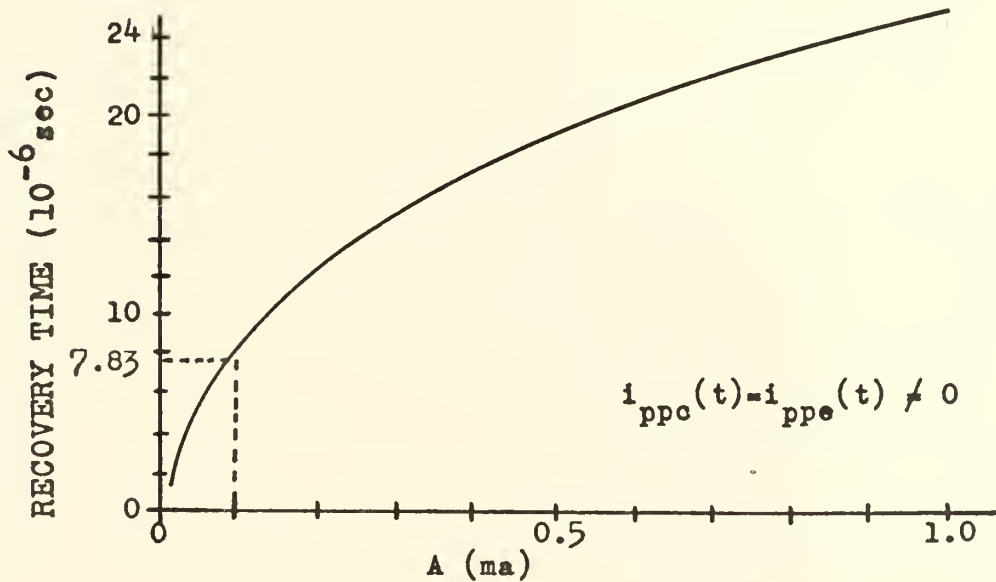
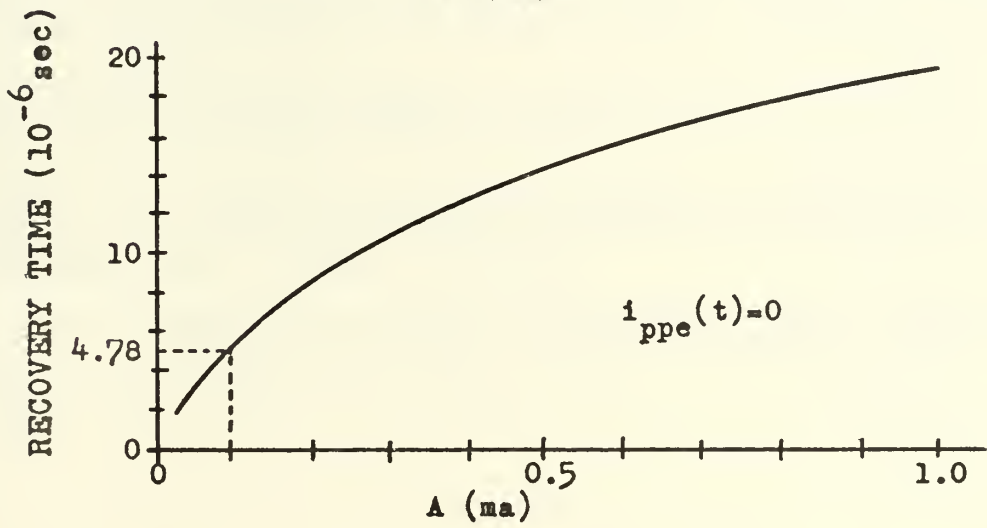
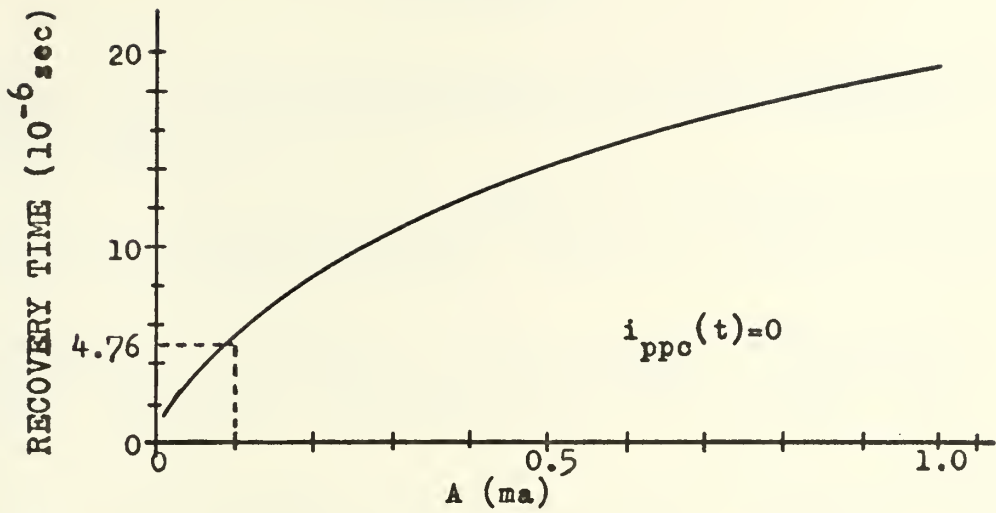


FIG. 5.7 RECOVERY TIME VS. A

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| 13. ABSTRACT <p>This thesis is concerned with the numerical solution of a semiconductor junction recovery from a radiation pulse. The junction is represented by an Ebers-Moll model to account for diffusion current and space-charge capacitance. The radiation pulse is considered as giving rise to a photocurrent to which it is related by a linear differential equation. Exact solutions are presented and the recovery time is presented and discussed as a function of several parameters. A simple piecewise linear analysis for a diode circuit is also presented to provide insight into the nature of the transient response and the recovery time.</p> | | | |

| KEY WORDS | LINK A | | LINK B | | LINK C | |
|------------------------|--------|----|--------|----|--------|----|
| | ROLE | WT | ROLE | WT | ROLE | WT |
| Radiation Effect | | | | | | |
| Semiconductor Junction | | | | | | |
| Mathematical Model | | | | | | |
| Sensitivity Function | | | | | | |

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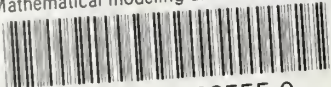
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